

# 第5章 超場

## カイラル超場

この超対称性変換と共変微分操作は交換可能なので、共変微分演算は超対称性不変な操作になっている。そのため、

$$D_\alpha \Phi(x, \theta, \bar{\theta}) = 0 \text{ や } \bar{D}_\alpha \Phi(x, \theta, \bar{\theta}) = 0$$

のような拘束条件を課することができる。特に、

$$\bar{D}_\alpha \Phi(x, \theta, \bar{\theta}) = 0 \left( \Leftarrow \frac{\partial}{\partial \bar{\theta}^\alpha} \Phi(x, \theta, \bar{\theta}) = 0 \right)$$

を満たす超場を

カイラル超場

という。 $\bar{\theta}$ に依存しない場の拡張定義である。

$$\bar{D}_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha \frac{\partial}{\partial x^\mu}$$

を用いると、

$$\begin{aligned} \bar{D}_\alpha y &= \left( -\frac{\partial}{\partial \bar{\theta}^\alpha} + i(\theta\sigma^\nu)_\alpha \frac{\partial}{\partial x^\nu} \right) (x^\mu - i\theta\sigma^\mu \bar{\theta}) = -(i\theta\sigma^\mu)_\alpha + i(\theta\sigma^\mu)_\alpha \eta_\nu^\mu = 0 \\ &\text{with } y^\mu = x^\mu - i\theta\sigma^\mu \bar{\theta} \\ \bar{D}_\alpha \theta &= 0 \end{aligned}$$

なので、 $y^\mu (= x^\mu - i\theta\sigma^\mu \bar{\theta})$ と $\theta$ の関数は自動的にカイラル超場になる：

$$\boxed{\bar{D}_\alpha \Phi(y, \theta) = 0 \text{ with } y^\mu = x^\mu - i\theta\sigma^\mu \bar{\theta}}$$

そこで、新たに $y$ を用いて、超変換演算子と共変微分を書き換えると、

$$\begin{aligned} \tilde{S}_\alpha &= \frac{\partial}{\partial \theta^\alpha} + i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial \theta^\alpha} + \left[ \frac{\partial y^\nu}{\partial \theta^\alpha} + i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial y^\nu}{\partial x^\mu} \right] \frac{\partial}{\partial y^\nu} \\ &\Leftarrow y^\mu = x^\mu - i\theta\sigma^\mu \bar{\theta} \rightarrow \frac{\partial y^\nu}{\partial \theta^\alpha} = -i(\sigma^\nu \bar{\theta})_\alpha \\ &= \frac{\partial}{\partial \theta^\alpha} + \left[ -i(\sigma^\nu \bar{\theta})_\alpha + i(\sigma^\mu \bar{\theta})_\alpha \eta_\mu^\nu \right] \frac{\partial}{\partial y^\nu} = \frac{\partial}{\partial \theta^\alpha} \\ \bar{\tilde{S}}_\alpha &= -\frac{\partial}{\partial \bar{\theta}^\alpha} - i(\theta\sigma^\mu)_\alpha \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial \bar{\theta}^\alpha} + \left[ -\frac{\partial y^\nu}{\partial \bar{\theta}^\alpha} - i(\theta\sigma^\mu)_\alpha \frac{\partial y^\nu}{\partial x^\mu} \right] \frac{\partial}{\partial y^\nu} \\ &\Leftarrow y^\mu = x^\mu - i\theta\sigma^\mu \bar{\theta} \rightarrow \frac{\partial y^\nu}{\partial \bar{\theta}^\alpha} = i(\theta\sigma^\mu)_\alpha \\ &= -\frac{\partial}{\partial \bar{\theta}^\alpha} + \left[ -i(\theta\sigma^\mu)_\alpha - i(\theta\sigma^\mu)_\alpha \eta_\mu^\nu \right] \frac{\partial}{\partial y^\nu} \\ &= -\frac{\partial}{\partial \bar{\theta}^\alpha} - 2i(\theta\sigma^\mu)_\alpha \frac{\partial}{\partial y^\nu} \end{aligned}$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial \theta^\alpha} - 2i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial y^\nu}$$

$$\bar{D}_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} + i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial x^\mu} = -\frac{\partial}{\partial \bar{\theta}^\alpha}$$

故に、

$$\left. \begin{array}{l} \tilde{S}_\alpha = \frac{\partial}{\partial \theta^\alpha}, \quad \bar{\tilde{S}}_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} - 2i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial y^\nu} \\ D_\alpha = \frac{\partial}{\partial \theta^\alpha} - 2i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial y^\nu}, \quad \bar{D}_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} \end{array} \right\} \text{acting on } \Phi(x - i\theta \sigma \bar{\theta}, \theta, \bar{\theta})$$

さて、カイラル超場は  $\Phi(y, \theta)$ 、3つの成分がある。

$$\Phi(y, \theta) = a\phi(y) + b\theta\psi(y) + cF(y)\theta\theta \quad \text{with } y = x - i\theta\sigma\bar{\theta}$$

これと、

$$\tilde{S}_\alpha = \frac{\partial}{\partial \theta^\alpha}, \quad \bar{\tilde{S}}_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} - 2i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial y^\nu} \quad \text{with } y = x - i\theta\sigma\bar{\theta}$$

$$\delta\Phi(y) = [\varepsilon S + \bar{\varepsilon} \bar{S}, \Phi(y)] = -(\varepsilon \tilde{S} + \bar{\varepsilon} \bar{\tilde{S}})\Phi(y)$$

$$[\varepsilon_1 S, \bar{\varepsilon}_2 \bar{S}] = 2\varepsilon_1 \sigma^\mu \bar{\varepsilon}_2 P_\mu \quad \& \quad [\varepsilon_1 \tilde{S}, \bar{\varepsilon}_2 \bar{\tilde{S}}] = 2\varepsilon_1 \sigma^\mu \bar{\varepsilon}_2 \tilde{P}_\mu$$

および、

$$\delta\phi = \sqrt{2}\varepsilon\psi, \quad \delta F = \sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\varepsilon}, \quad \delta\psi_\alpha = -\sqrt{2}i(\sigma^\mu\bar{\varepsilon})_\alpha\partial_\mu\phi + \sqrt{2}\varepsilon_\alpha F$$

を用いて、超場の成分の係数  $a, b, c$  をもとめる。

$$\begin{aligned} \delta\Phi &= a\delta\phi(y) + b\theta\delta\psi(y) + c\delta F(y)\theta\theta \\ &= a\sqrt{2}\varepsilon\psi + b\theta^\alpha \left( -\sqrt{2}i(\sigma^\mu\bar{\varepsilon})_\alpha \partial_\mu\phi + \sqrt{2}\varepsilon_\alpha F \right) + c\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\varepsilon}\theta\theta \\ &= a\sqrt{2}\varepsilon\psi - b\sqrt{2}i(\theta\sigma^\mu\bar{\varepsilon})\partial_\mu\phi + b\sqrt{2}\theta\varepsilon F + c\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\varepsilon}\theta\theta \end{aligned}$$

一方、

$$\begin{aligned} \delta\Phi &= [\varepsilon S + \bar{\varepsilon} \bar{S}, \Phi] = -(\varepsilon \tilde{S} + \bar{\varepsilon} \bar{\tilde{S}})\Phi = -(\varepsilon \tilde{S} + \bar{\varepsilon} \bar{\tilde{S}})(a\phi(y) + b\theta\psi(y) + cF(y)\theta\theta) \\ &= -\left[ \varepsilon^\alpha \frac{\partial}{\partial \theta^\alpha} - \bar{\varepsilon}^\alpha \left( -\frac{\partial}{\partial \bar{\theta}^\alpha} - 2i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial y^\mu} \right) \right] (a\phi(y) + b\theta\psi(y) + cF(y)\theta\theta) \\ &= -\left( \varepsilon^\alpha \frac{\partial}{\partial \theta^\alpha} \right) (a\phi(y) + b\theta\psi(y) + cF(y)\theta\theta) + \left( -\bar{\varepsilon}^\alpha \frac{\partial}{\partial \bar{\theta}^\alpha} \right) (a\phi(y) + b\theta\psi(y) + cF(y)\theta\theta) \\ &\quad + \left( -2\bar{\varepsilon}^\alpha i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial y^\mu} \right) (a\phi(y) + b\theta\psi(y) + cF(y)\theta\theta) \\ &= -b\varepsilon\psi(y) - 2c\varepsilon\theta F(y) - \left( 2\bar{\varepsilon}^\alpha i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial y^\mu} \right) (a\phi(y) + b\theta\psi(y)) \end{aligned}$$

$$\begin{aligned}
&= -b\varepsilon\psi(y) - 2ci\varepsilon\theta F(y) - 2ia\bar{\varepsilon}^\alpha(\theta\sigma^\mu)_\alpha \frac{\partial}{\partial y^\mu}\phi(y) - 2ib(-\theta\sigma^\mu\bar{\varepsilon})\theta^\beta \frac{\partial}{\partial y^\mu}\psi_\beta(y) \\
&= -b\varepsilon\psi(y) - 2ci\varepsilon\theta F(y) + 2ia(\theta\sigma^\mu\bar{\varepsilon}) \frac{\partial}{\partial y^\mu}\phi(y) + 2ib\theta^\alpha(\sigma^\mu\bar{\varepsilon})_\alpha \left( -\theta_\beta \frac{\partial}{\partial y^\mu}\psi^\beta(y) \right) \\
&= -b\varepsilon\psi(y) - 2ci\varepsilon\theta F(y) + 2ia(\theta\sigma^\mu\bar{\varepsilon}) \frac{\partial}{\partial y^\mu}\phi(y) + 2ib\theta^\alpha\theta_\beta(\sigma^\mu\bar{\varepsilon})_\alpha \frac{\partial}{\partial y^\mu}\psi^\beta(y) \\
&\quad \Leftarrow \theta^\alpha\theta_\beta = \frac{1}{2}\delta_\beta^\alpha\theta\theta \\
&= -b\varepsilon\psi(y) - 2ci\varepsilon\theta F(y) + 2ia(\theta\sigma^\mu\bar{\varepsilon}) \frac{\partial}{\partial y^\mu}\phi(y) + 2ib\frac{1}{2}\delta_\beta^\alpha\theta\theta(\sigma^\mu\bar{\varepsilon})_\alpha \frac{\partial}{\partial y^\mu}\psi^\beta(y) \\
&= -b\varepsilon\psi(y) - 2ci\varepsilon\theta F(y) + 2ia(\theta\sigma^\mu\bar{\varepsilon}) \frac{\partial}{\partial y^\mu}\phi(y) - ib\theta\theta \left( \frac{\partial}{\partial y^\mu}\psi(y)\sigma^\mu\bar{\varepsilon} \right)
\end{aligned}$$

つまり、

$$\begin{aligned}
\delta\Phi &= a\sqrt{2}\varepsilon\psi - b\sqrt{2}i(\theta\sigma^\mu\bar{\varepsilon})\partial_\mu\phi + b\sqrt{2}\theta\varepsilon F + c\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\varepsilon}\theta\theta \\
&= -b\varepsilon\psi(y) - 2ci\varepsilon\theta F(y) + 2ia(\theta\sigma^\mu\bar{\varepsilon}) \frac{\partial}{\partial y^\mu}\phi(y) - ib\theta\theta \left( \frac{\partial}{\partial y^\mu}\psi(y)\sigma^\mu\bar{\varepsilon} \right)
\end{aligned}$$

より、

$$\begin{aligned}
\delta\Phi &= a\sqrt{2}\varepsilon\psi - b\sqrt{2}i(\theta\sigma^\mu\bar{\varepsilon})\partial_\mu\phi + b\sqrt{2}\theta\varepsilon F + c\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\varepsilon}\theta\theta \\
&= -b\varepsilon\psi(y) - 2c\varepsilon\theta F(y) + 2ia(\theta\sigma^\mu\bar{\varepsilon}) \frac{\partial}{\partial y^\mu}\phi(y) - ib\theta\theta \left( \frac{\partial}{\partial y^\mu}\psi(y)\sigma^\mu\bar{\varepsilon} \right) \\
\Rightarrow &\begin{cases} a\sqrt{2} = -b \Leftarrow \varepsilon\psi \\ -b\sqrt{2}i = 2ia \Leftarrow \partial_\mu\phi \\ b\sqrt{2} = -2c \Leftarrow F \\ c\sqrt{2}i = -ib \Leftarrow \partial_\mu\psi \end{cases} \rightarrow a=1, b=-\sqrt{2}, c=1
\end{aligned}$$

これから、

$$\Phi(y, \theta) = \phi(y) - \sqrt{2}\theta\psi(y) + F(y)\theta\theta \quad \text{with } y = x - i\theta\sigma\bar{\theta}$$

がカイラル超場になる。

## ベクトル超場

成分の変換性は「第3章 超対称性変換2」より

$$\delta V^\mu = -i(\varepsilon\sigma^\mu\bar{\lambda} + \bar{\varepsilon}\sigma^\mu\lambda), \quad \delta\lambda_\alpha = -\frac{i}{2}(\sigma^{\mu\nu}\varepsilon)_\alpha V_{\mu\nu}, \quad \delta\bar{\lambda}_\alpha = \frac{i}{2}(\bar{\varepsilon}\sigma^{\mu\nu})_\alpha V_{\mu\nu}$$

$$D_\mu\lambda = \partial_\mu\lambda + ig[V_\mu, \lambda], \quad D_\mu\bar{\lambda} = \partial_\mu\bar{\lambda} + ig[V_\mu, \bar{\lambda}], \quad V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]$$

である。ここで、 $\lambda$ の代わりに $\psi$ を用いることにする。

$$\begin{aligned}
V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta m(x) - i\bar{\theta}\bar{\theta}m^\dagger(x) + a\theta\sigma^\mu\bar{\theta}V_\mu \\
&\quad + ib[\theta\theta\bar{\theta}\bar{\psi}(x) - \bar{\theta}\bar{\theta}\theta\psi(x)] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}d(x) \quad (a, b \text{ を決める})
\end{aligned}$$

と成分表示をする。超場に対する超対称性変換演算子は、

$$\xi \tilde{S} = i\xi \sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} + \xi^\alpha \frac{\partial}{\partial \theta^\alpha}, \quad \bar{\xi} \tilde{S} = -i\theta \sigma^\mu \bar{\xi} \frac{\partial}{\partial x^\mu} + \bar{\xi}^\alpha \frac{\partial}{\partial \bar{\theta}^\alpha}$$

なので

$$\begin{aligned} \delta V(x, \theta, \bar{\theta}) &= [\varepsilon S + \bar{\varepsilon} \bar{S}_\mu, V(x, \theta, \bar{\theta})] = -(\varepsilon \tilde{S} + \bar{\varepsilon} \tilde{S}) V(x, \theta, \bar{\theta}) \\ &= -\left( i\xi \sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} + \xi^\alpha \frac{\partial}{\partial \theta^\alpha} - i\theta \sigma^\mu \bar{\xi} \frac{\partial}{\partial x^\mu} + \bar{\xi}^\alpha \frac{\partial}{\partial \bar{\theta}^\alpha} \right) V(x, \theta, \bar{\theta}) \end{aligned}$$

これから、

$$\begin{aligned} \delta V(x, \theta, \bar{\theta}) &= \delta(a\theta \sigma^\mu \bar{\theta} V_\mu + b[i\theta \theta \bar{\theta} \bar{\psi}(x) - i\bar{\theta} \bar{\theta} \theta \psi(x)] + \dots) \\ &= -\left( i\varepsilon \sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} + \varepsilon^\alpha \frac{\partial}{\partial \theta^\alpha} - i\theta \sigma^\mu \bar{\varepsilon} \frac{\partial}{\partial x^\mu} + \bar{\varepsilon}^\alpha \frac{\partial}{\partial \bar{\theta}^\alpha} \right) (a\theta \sigma^\nu \bar{\theta} V_\nu + ib[\theta \theta \bar{\theta} \bar{\psi}(x) - \bar{\theta} \bar{\theta} \theta \psi(x)] + \dots) \\ &= -\left( i\varepsilon \sigma^\mu \bar{\theta} \left( \frac{\partial}{\partial x^\mu} a\theta \sigma^\nu \bar{\theta} V_\nu \right) + ib\varepsilon^\alpha \frac{\partial}{\partial \theta^\alpha} \theta \theta \bar{\theta} \bar{\psi}(x) - i\theta \sigma^\mu \bar{\varepsilon} \frac{\partial}{\partial x^\mu} (a\theta \sigma^\nu \bar{\theta} V_\nu) - ib\bar{\varepsilon}^\alpha \frac{\partial}{\partial \bar{\theta}^\alpha} \bar{\theta} \bar{\theta} \theta \psi(x) + \dots \right) \\ &= -i\varepsilon \sigma^\mu \bar{\theta} \left( \frac{\partial}{\partial x^\mu} a\theta \sigma^\nu \bar{\theta} V_\nu \right) - 2ib\varepsilon \theta \bar{\theta} \bar{\psi}(x) + i\theta \sigma^\mu \bar{\varepsilon} \frac{\partial}{\partial x^\mu} (a\theta \sigma^\nu \bar{\theta} V_\nu) + 2bi\bar{\varepsilon} \bar{\theta} \theta \psi(x) + \dots \\ &= -i\varepsilon \sigma^\mu \bar{\theta} \left( -\frac{\partial}{\partial x^\mu} a\bar{\theta} \bar{\sigma}^\nu \theta V_\nu \right) - 2bi\varepsilon \theta \bar{\theta} \bar{\psi}(x) + i\theta \sigma^\mu \bar{\varepsilon} \frac{\partial}{\partial x^\mu} (-a\bar{\theta} \bar{\sigma}^\nu \theta V_\nu) + 2bi\bar{\varepsilon} \bar{\theta} \theta \psi(x) + \dots \\ &= ia\varepsilon \sigma^\mu \bar{\theta} \cdot \bar{\theta} \bar{\sigma}^\nu \theta \left( \frac{\partial}{\partial x^\mu} V_\nu \right) - 2bi\varepsilon \theta \cdot \bar{\theta} \bar{\psi}(x) - ai\bar{\theta} \bar{\sigma}^\nu \theta \cdot \theta \sigma^\mu \bar{\varepsilon} \frac{\partial}{\partial x^\mu} V_\nu + 2bi\bar{\varepsilon} \bar{\theta} \cdot \theta \psi(x) + \dots \\ &\quad \Leftarrow \theta^\alpha \bar{\theta}^\beta = \frac{1}{2} (\bar{\sigma}_\mu)^\beta{}_\alpha \theta^\mu \bar{\theta}^\beta, \bar{\theta}_\alpha \theta_\beta = \frac{1}{2} (\sigma_\mu)_{\beta\alpha} \bar{\theta}^\mu \theta_\beta, \theta^\alpha \theta_\beta = \frac{1}{2} \delta_\beta^\alpha \theta \theta, \bar{\theta}_\alpha \bar{\theta}^\beta = \frac{1}{2} \delta_\alpha^\beta \bar{\theta} \bar{\theta} \\ &= -\frac{1}{2} ia\bar{\theta} \bar{\theta} \cdot \varepsilon \sigma^\mu \bar{\sigma}^\nu \theta \left( \frac{\partial}{\partial x^\mu} V_\nu \right) - 2bi\varepsilon^\beta \left( -\frac{1}{2} (\sigma_\mu)_{\beta\alpha} \bar{\theta} \bar{\sigma}^\mu \theta \right) \bar{\psi}^\alpha(x) + \frac{1}{2} ai\bar{\theta} \bar{\sigma}^\nu \sigma^\mu \bar{\varepsilon} \cdot \theta \theta \frac{\partial}{\partial x^\mu} V_\nu \\ &\quad + 2bi\bar{\varepsilon}_\beta \left( -\frac{1}{2} (\bar{\sigma}_\mu)^\beta{}_\alpha \theta \sigma^\mu \bar{\theta} \right) \psi^\alpha(x) + \dots \Leftarrow \varepsilon \sigma^\mu \bar{\sigma}^\nu \theta = \theta \sigma^\nu \bar{\sigma}^\mu \varepsilon, \bar{\theta} \bar{\sigma}^\nu \sigma^\mu \bar{\varepsilon} = \bar{\varepsilon} \bar{\sigma}^\mu \sigma^\nu \bar{\theta} \\ &= -\frac{1}{2} ia\bar{\theta} \bar{\theta} \cdot \theta \sigma^\nu \bar{\sigma}^\mu \varepsilon \left( \frac{\partial}{\partial x^\mu} V_\nu \right) + bi\varepsilon \sigma_\mu \bar{\psi}(x) \bar{\theta} \bar{\sigma}^\mu \theta + \frac{1}{2} ai\bar{\varepsilon} \bar{\sigma}^\mu \sigma^\nu \bar{\theta} \cdot \theta \theta \frac{\partial}{\partial x^\mu} V_\nu - bi\bar{\varepsilon} \bar{\sigma}_\mu \psi(x) \theta \sigma^\mu \bar{\theta} + \dots \\ &= -\frac{1}{2} ia\bar{\theta} \bar{\theta} \cdot \theta (g^{\mu\nu} + i\sigma^{\nu\mu}) \varepsilon \left( \frac{\partial}{\partial x^\mu} V_\nu \right) + bi\varepsilon \sigma_\mu \bar{\psi}(x) \bar{\theta} \bar{\sigma}^\mu \theta + \frac{1}{2} ai\theta \bar{\theta} \cdot \bar{\varepsilon} (g^{\mu\nu} + i\bar{\sigma}^{\mu\nu}) \bar{\theta} \frac{\partial}{\partial x^\mu} V_\nu \\ &\quad - bi\bar{\varepsilon} \bar{\sigma}_\mu \psi(x) \theta \sigma^\mu \bar{\theta} + \dots \\ &= -\frac{1}{2} ia\bar{\theta} \bar{\theta} \cdot \left( \varepsilon \theta \frac{\partial V^\mu}{\partial x^\mu} - i\theta \sigma^{\mu\nu} \varepsilon \frac{\partial V_\nu}{\partial x^\mu} \right) + bi(-\varepsilon \sigma_\mu \bar{\psi}(x) - \bar{\varepsilon} \bar{\sigma}_\mu \psi(x)) \theta \sigma^\mu \bar{\theta} + \frac{1}{2} ai\theta \bar{\theta} \left( \bar{\theta} \bar{\varepsilon} \frac{\partial V^\mu}{\partial x^\mu} - i\bar{\varepsilon} \bar{\sigma}^{\nu\mu} \bar{\theta} \frac{\partial V_\nu}{\partial x^\mu} \right) \\ &\Rightarrow \begin{cases} \delta(a\theta \sigma^\mu \bar{\theta} V_\mu) = bi(-\varepsilon \sigma_\mu \bar{\psi}(x) - \bar{\varepsilon} \bar{\sigma}_\mu \psi(x)) \theta \sigma^\mu \bar{\theta} + \dots \\ \delta(bi\theta \theta \bar{\theta} \bar{\psi}(x)) = \frac{1}{2} ai\theta \bar{\theta} \left( \bar{\theta} \bar{\varepsilon} \frac{\partial V^\mu}{\partial x^\mu} - i\bar{\varepsilon}_\beta (\bar{\sigma}^{\nu\mu})^\beta{}_\alpha \bar{\theta}^\alpha \frac{\partial V_\nu}{\partial x^\mu} \right) + \dots \\ \delta(-bi\bar{\theta} \bar{\theta} \theta \psi(x)) = -\frac{1}{2} ia\bar{\theta} \bar{\theta} \cdot \left( \varepsilon \theta \frac{\partial V^\mu}{\partial x^\mu} - i\theta^\alpha (\sigma^{\mu\nu})^\beta{}_{\bar{\alpha}} \varepsilon^\beta \frac{\partial V_\nu}{\partial x^\mu} \right) + \dots \end{cases} \end{aligned}$$

故に、

$$\begin{aligned}
 a\delta V_\mu &= -bi(\varepsilon\sigma_\mu\bar{\psi}(x) + \bar{\varepsilon}\bar{\sigma}_\mu\psi(x)) + \dots \Leftarrow \delta V^\mu = -i(\varepsilon\sigma^\mu\bar{\psi} + \bar{\varepsilon}\bar{\sigma}^\mu\psi) \\
 -bi\delta\bar{\psi}_\alpha(x) &= \frac{1}{2}ai\left(\bar{\varepsilon}_\alpha\frac{\partial V^\mu}{\partial x^\mu} - i\bar{\varepsilon}_\beta(\bar{\sigma}^{\nu\mu})_\alpha^\beta\frac{\partial V_\nu}{\partial x^\mu}\right) + \dots \Leftarrow \delta\bar{\psi}_\alpha = \frac{i}{2}(\bar{\varepsilon}\bar{\sigma}^{\mu\nu})_\alpha V_{\mu\nu} \\
 -bi\delta\psi_\alpha(x) &= -\frac{1}{2}ia\left(\varepsilon_\alpha\frac{\partial V^\mu}{\partial x^\mu} - i(\sigma^{\mu\nu})_\alpha^\beta\varepsilon_\beta\frac{\partial V_\nu}{\partial x^\mu}\right) + \dots \Leftarrow \delta\psi_\alpha = -\frac{i}{2}(\sigma^{\mu\nu}\varepsilon)_\alpha V_{\mu\nu}
 \end{aligned}$$

より、 $a=b=1$ を得る。まとめて、

$$\begin{aligned}
 V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta m(x) - i\bar{\theta}\bar{\theta}m^\dagger(x) + \theta\sigma^\mu\bar{\theta}V_\mu \\
 &\quad + i\left[\theta\theta\bar{\theta}\bar{\psi}(x) - \bar{\theta}\bar{\theta}\theta\psi(x)\right] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}d(x) \\
 \delta V(x, \theta, \bar{\theta}) &= \left[\varepsilon S + \bar{\varepsilon}\bar{S}_\mu, V(x, \theta, \bar{\theta})\right] = -\left(i\varepsilon\sigma^\mu\bar{\theta}\frac{\partial}{\partial x^\mu} + \varepsilon^\alpha\frac{\partial}{\partial\theta^\alpha} - i\theta\sigma^\mu\bar{\varepsilon}\frac{\partial}{\partial x^\mu} + \bar{\varepsilon}^\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\right)V(x, \theta, \bar{\theta})
 \end{aligned}$$

である。

次に、すべての成分場の変換性を求める。

$$\begin{aligned}
 \delta V(x, \theta, \bar{\theta}) &= \delta C(x) + i\theta\delta\chi(x) - i\bar{\theta}\delta\bar{\chi}(x) + i\theta\theta\delta m(x) - i\bar{\theta}\bar{\theta}\delta m^\dagger(x) + \theta\sigma^\mu\bar{\theta}\delta V_\mu \\
 &\quad + i\left[\theta\theta\bar{\theta}\delta\bar{\psi}(x) - \bar{\theta}\bar{\theta}\theta\delta\psi(x)\right] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\delta d(x) \\
 &= -\left(i\varepsilon\sigma^\mu\bar{\theta}\frac{\partial}{\partial x^\mu} + \varepsilon^\alpha\frac{\partial}{\partial\theta^\alpha} - i\theta\sigma^\mu\bar{\varepsilon}\frac{\partial}{\partial x^\mu} + \bar{\varepsilon}^\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\right) \\
 &\quad \left[ C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta m(x) - i\bar{\theta}\bar{\theta}m^\dagger(x) + \theta\sigma^\mu\bar{\theta}V_\mu \right. \\
 &\quad \left. + i\left[\theta\theta\bar{\theta}\bar{\psi}(x) - \bar{\theta}\bar{\theta}\theta\psi(x)\right] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}d(x) \right]
 \end{aligned}$$

$$(*)\theta \& \bar{\theta} \text{無し} : \delta C(x) = -\left(\varepsilon^\alpha\frac{\partial i\theta\chi(x)}{\partial\theta^\alpha} - i\bar{\varepsilon}^\alpha\frac{\partial i\bar{\theta}\bar{\chi}(x)}{\partial\bar{\theta}^\alpha}\right) = -i\left(\varepsilon^\alpha\frac{\partial\theta\chi(x)}{\partial\theta^\alpha} - \bar{\varepsilon}^\alpha\frac{\partial\bar{\theta}\bar{\chi}(x)}{\partial\bar{\theta}^\alpha}\right) = -i(\varepsilon\chi(x) - \bar{\varepsilon}\bar{\chi}(x))$$

$$\begin{aligned}
 (*)\theta \& \bar{\theta} : i\theta\delta\chi(x) - i\bar{\theta}\delta\bar{\chi}(x) &= -\left(\begin{aligned} &i\varepsilon\sigma^\mu\bar{\theta}\frac{\partial C(x)}{\partial x^\mu} + \varepsilon^\alpha\frac{\partial(\theta\sigma^\mu\bar{\theta}V_\mu + i\theta\theta m(x))}{\partial\theta^\alpha} \\ &-i\theta\sigma^\mu\bar{\varepsilon}\frac{\partial C(x)}{\partial x^\mu} + \bar{\varepsilon}^\alpha\frac{\partial(\theta\sigma^\mu\bar{\theta}V_\mu - i\bar{\theta}\bar{\theta}m^\dagger(x))}{\partial\bar{\theta}^\alpha} \end{aligned}\right) \\
 &= -\left(i\varepsilon\sigma^\mu\bar{\theta}\frac{\partial C(x)}{\partial x^\mu} + \varepsilon\sigma^\mu\bar{\theta}V_\mu + 2i\varepsilon\theta m(x) - i\theta\sigma^\mu\bar{\varepsilon}\frac{\partial C(x)}{\partial x^\mu} + \theta\sigma^\mu\bar{\varepsilon}V_\mu - 2i\bar{\varepsilon}\bar{\theta}m^\dagger(x)\right) \\
 &= -\theta^\alpha\left((\sigma^\mu\bar{\varepsilon})_\alpha V_\mu + 2i\varepsilon_\alpha m(x) - i(\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial C(x)}{\partial x^\mu}\right) - \left((\varepsilon\sigma^\mu)_\alpha V_\mu - 2i\bar{\varepsilon}_\alpha m^\dagger(x) + i(\varepsilon\sigma^\mu)_\alpha\frac{\partial C(x)}{\partial x^\mu}\right)\bar{\theta}^\alpha \\
 \Rightarrow \delta\chi_\alpha(x) &= -\left(2\varepsilon_\alpha m(x) - i(\sigma^\mu\bar{\varepsilon})_\alpha V_\mu - (\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial C(x)}{\partial x^\mu}\right) \\
 \delta\bar{\chi}_\alpha(x) &= -\left(2\bar{\varepsilon}_\alpha m^\dagger(x) + i(\varepsilon\sigma^\mu)_\alpha V_\mu - (\varepsilon\sigma^\mu)_\alpha\frac{\partial C(x)}{\partial x^\mu}\right)
 \end{aligned}$$

(\*)  $\theta\theta$  &  $\bar{\theta}\bar{\theta}$ 

$$\begin{aligned}
i\theta\theta\delta m(x) - i\bar{\theta}\bar{\theta}\delta m^\dagger(x) &= -\left( i\varepsilon\sigma^\mu\bar{\theta} \frac{\partial(-i\bar{\theta}\bar{\chi}(x))}{\partial x^\mu} + \varepsilon^\alpha \frac{\partial(-i\bar{\theta}\bar{\theta}\theta\psi(x))}{\partial\theta^\alpha} - i\theta\sigma^\mu\bar{\varepsilon} \frac{\partial i\theta\chi(x)}{\partial x^\mu} + \bar{\varepsilon}^\alpha \frac{\partial i\theta\theta\bar{\theta}\bar{\psi}(x)}{\partial\bar{\theta}^\alpha} \right) \\
&= -\left( -ii\varepsilon\sigma^\mu\bar{\theta} \cdot \bar{\theta} \frac{\partial\bar{\chi}(x)}{\partial x^\mu} - i\varepsilon\lambda(x)\bar{\theta}\bar{\theta} - ii \frac{\partial\chi(x)}{\partial x^\mu} \theta \cdot \theta\sigma^\mu\bar{\varepsilon} + i\bar{\varepsilon}\bar{\lambda}(x)\theta\theta \right) \\
&= -\left( (\varepsilon\sigma^\mu)_\beta \bar{\theta}^\beta \cdot \bar{\theta}_\alpha \frac{\partial\bar{\chi}^\alpha(x)}{\partial x^\mu} + \frac{\partial\chi^\beta(x)}{\partial x^\mu} \theta_\beta \cdot \theta^\alpha (\sigma^\mu\bar{\varepsilon})_\alpha - i\varepsilon\psi(x)\bar{\theta}\bar{\theta} + i\bar{\varepsilon}\bar{\psi}(x)\theta\theta \right) \\
&\Leftarrow \theta^\alpha\theta_\beta = \frac{1}{2}\delta_\beta^\alpha\theta\theta, \bar{\theta}_\alpha\bar{\theta}^\beta = \frac{1}{2}\delta_\alpha^\beta\bar{\theta}\bar{\theta} \\
&= -\left( (\varepsilon\sigma^\mu)_\beta \left( -\frac{1}{2}\delta_\alpha^\beta\bar{\theta}\bar{\theta} \right) \frac{\partial\bar{\chi}^\alpha(x)}{\partial x^\mu} + \frac{\partial\chi^\beta(x)}{\partial x^\mu} \left( -\frac{1}{2}\delta_\beta^\alpha\theta\theta \right) (\sigma^\mu\bar{\varepsilon})_\alpha - i\varepsilon\psi(x)\bar{\theta}\bar{\theta} + i\bar{\varepsilon}\bar{\psi}(x)\theta\theta \right) \\
&= -\left( -\frac{1}{2}\varepsilon\sigma^\mu \frac{\partial\bar{\chi}(x)}{\partial x^\mu} \bar{\theta}\bar{\theta} - \frac{1}{2} \frac{\partial\chi(x)}{\partial x^\mu} \sigma^\mu\bar{\varepsilon}\theta\theta - i\varepsilon\psi(x)\bar{\theta}\bar{\theta} + i\bar{\varepsilon}\bar{\psi}(x)\theta\theta \right) \\
&= \frac{1}{2}\varepsilon\sigma^\mu \frac{\partial\bar{\chi}(x)}{\partial x^\mu} \bar{\theta}\bar{\theta} + i\varepsilon\psi(x)\bar{\theta}\bar{\theta} + \frac{1}{2} \frac{\partial\chi(x)}{\partial x^\mu} \sigma^\mu\bar{\varepsilon}\theta\theta - i\bar{\varepsilon}\bar{\psi}(x)\theta\theta \\
&\Rightarrow \delta m(x) = -\bar{\varepsilon}\bar{\psi}(x) - i \frac{1}{2} \frac{\partial\chi(x)}{\partial x^\mu} \sigma^\mu\bar{\varepsilon}, \quad \delta m^\dagger(x) = -\varepsilon\psi(x) + i \frac{1}{2} \varepsilon\sigma^\mu \frac{\partial\bar{\chi}(x)}{\partial x^\mu}
\end{aligned}$$

$$\begin{aligned}
(*) \theta\bar{\theta} : \theta\sigma^\mu\bar{\theta}\delta V_\mu &= -\left( i\varepsilon\sigma^\mu\bar{\theta} \frac{\partial i\theta\chi(x)}{\partial x^\mu} + \varepsilon^\alpha \frac{\partial i\theta\theta\bar{\theta}\bar{\psi}(x)}{\partial\theta^\alpha} - i\theta\sigma^\mu\bar{\varepsilon} \frac{\partial(-i\bar{\theta}\bar{\chi}(x))}{\partial x^\mu} + \bar{\varepsilon}^\alpha \frac{\partial(-i\bar{\theta}\bar{\theta}\theta\psi(x))}{\partial\bar{\theta}^\alpha} \right) \\
&= -\left( ii(\varepsilon\sigma^\mu)_\beta \bar{\theta}^\beta\theta^\alpha \frac{\partial\chi_\alpha(x)}{\partial x^\mu} - i(-i)(-\bar{\theta}^\beta)\theta^\alpha (\sigma^\mu\bar{\varepsilon})_\alpha \frac{\partial\bar{\chi}_\beta(x)}{\partial x^\mu} + \dots \right) \Leftarrow \theta^\alpha\bar{\theta}^\beta = \frac{1}{2}(\bar{\sigma}^\nu)^{\beta\alpha} \theta\sigma_\nu\bar{\theta} \\
&= -\left( ii(\varepsilon\sigma^\mu)_\beta \frac{\partial\chi_\alpha(x)}{\partial x^\mu} + i(-i)(\sigma^\mu\bar{\varepsilon})_\alpha \frac{\partial\bar{\chi}_\beta(x)}{\partial x^\mu} \right) \left( -\frac{1}{2}(\bar{\sigma}^\nu)^{\beta\alpha} \theta\sigma_\nu\bar{\theta} \right) + \dots \\
&= -\frac{1}{2} \left( \varepsilon\sigma^\mu\bar{\sigma}^\nu \frac{\partial\chi(x)}{\partial x^\mu} + \frac{\partial\bar{\chi}(x)}{\partial x^\mu} \bar{\sigma}^\nu\sigma^\mu\bar{\varepsilon} \right) \theta\sigma_\nu\bar{\theta} + \dots \\
&\Rightarrow \delta V_\mu = -\frac{1}{2} \left( \varepsilon\sigma^\nu\bar{\sigma}^\mu \frac{\partial\chi(x)}{\partial x^\nu} + \frac{\partial\bar{\chi}(x)}{\partial x^\nu} \bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon} \right) + \dots \\
&\Rightarrow \delta V_\mu = -i(\varepsilon\sigma^\mu\bar{\psi} + \bar{\varepsilon}\bar{\sigma}^\mu\psi) - \frac{1}{2} \left( \varepsilon\sigma^\nu\bar{\sigma}^\mu \frac{\partial\chi(x)}{\partial x^\nu} + \frac{\partial\bar{\chi}(x)}{\partial x^\nu} \bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon} \right)
\end{aligned}$$

(\*)  $\theta\theta\bar{\theta}$  &  $\bar{\theta}\bar{\theta}\theta$ 

$$i\theta\theta\bar{\theta}\delta\bar{\psi}(x) - i\bar{\theta}\bar{\theta}\theta\delta\psi(x) = -\left( i\varepsilon\sigma^\mu\bar{\theta} \frac{\partial(i\theta\theta m(x) + \theta\sigma^\nu\bar{\theta}V_\nu)}{\partial x^\mu} + \varepsilon^\alpha \frac{1}{2} \frac{\partial\theta\theta\bar{\theta}\bar{\theta}d(x)}{\partial\theta^\alpha} \right. \\
\left. - i\theta\sigma^\mu\bar{\varepsilon} \frac{\partial(-i\bar{\theta}\bar{\theta}m^\dagger(x) + \theta\sigma^\nu\bar{\theta}V_\nu)}{\partial x^\mu} - \bar{\varepsilon}^\alpha \frac{1}{2} \frac{\partial\bar{\theta}\bar{\theta}\theta\theta d(x)}{\partial\bar{\theta}^\alpha} \right)$$

$$\begin{aligned}
&= -ii\varepsilon\sigma^\mu\bar{\theta}\cdot\theta\theta\frac{\partial m(x)}{\partial x^\mu} - i\varepsilon\sigma^\mu\bar{\theta}\cdot\theta\sigma^\nu\bar{\theta}\frac{\partial V_\nu}{\partial x^\mu} - \varepsilon\theta\cdot\bar{\theta}\theta d(x) \\
&\quad + i(-i)\theta\sigma^\mu\bar{\varepsilon}\cdot\bar{\theta}\theta\frac{\partial m^\dagger(x)}{\partial x^\mu} + i\theta\sigma^\mu\bar{\varepsilon}\cdot\theta\sigma^\nu\bar{\theta}\frac{\partial V_\nu}{\partial x^\mu} + \bar{\varepsilon}\bar{\theta}\cdot\theta\theta d(x) \\
&= -i\boxed{\varepsilon\sigma^\mu\bar{\theta}\cdot\theta\sigma^\nu\bar{\theta}}\frac{\partial V_\nu}{\partial x^\mu} + i\boxed{\theta\sigma^\mu\bar{\varepsilon}\cdot\theta\sigma^\nu\bar{\theta}}\frac{\partial V_\nu}{\partial x^\mu} \\
&\quad + \varepsilon\sigma^\mu\bar{\theta}\cdot\theta\theta\frac{\partial m(x)}{\partial x^\mu} + \bar{\varepsilon}\bar{\theta}\cdot\theta\theta d(x) + \theta\sigma^\mu\bar{\varepsilon}\cdot\bar{\theta}\theta\frac{\partial m^\dagger(x)}{\partial x^\mu} - \varepsilon\theta\cdot\bar{\theta}\theta d(x) \\
&\Leftarrow \theta^\alpha\theta_\beta = \frac{1}{2}\delta_\beta^\alpha\theta\theta, \bar{\theta}_\alpha\bar{\theta}^\beta = \frac{1}{2}\delta_\alpha^\beta\bar{\theta}\bar{\theta} \\
&\quad \boxed{\varepsilon\sigma^\mu\bar{\theta}\cdot\theta\sigma^\nu\bar{\theta}} = -\varepsilon\sigma^\mu\bar{\theta}\cdot\bar{\theta}\bar{\sigma}^\nu\theta = -(\varepsilon\sigma^\mu)_\beta\bar{\theta}^\beta\cdot\bar{\theta}_\alpha(\bar{\sigma}^\nu\theta)^\alpha = \frac{1}{2}(\varepsilon\sigma^\mu)_\beta\delta_\alpha^\beta\bar{\theta}\bar{\theta}(\bar{\sigma}^\nu\theta)^\alpha = \frac{1}{2}\varepsilon\sigma^\mu\bar{\sigma}^\nu\theta\cdot\bar{\theta}\bar{\theta} \\
&\quad \boxed{\theta\sigma^\mu\bar{\varepsilon}\cdot\theta\sigma^\nu\bar{\theta}} = -\bar{\theta}\bar{\sigma}^\nu\theta\cdot\theta\sigma^\mu\bar{\varepsilon} = -(\bar{\theta}\bar{\sigma}^\nu)^\beta\theta_\beta\cdot\theta^\alpha(\sigma^\mu\bar{\varepsilon})_\alpha = \frac{1}{2}(\bar{\theta}\bar{\sigma}^\nu)^\beta\delta_\beta^\alpha\theta\theta(\sigma^\mu\bar{\varepsilon})_\alpha = \frac{1}{2}\bar{\theta}\bar{\sigma}^\nu\sigma^\mu\bar{\varepsilon}\cdot\theta\theta \\
&= -i\boxed{\frac{1}{2}\varepsilon\sigma^\mu\bar{\sigma}^\nu\theta\cdot\bar{\theta}\bar{\theta}}\frac{\partial V_\nu}{\partial x^\mu} + i\boxed{\frac{1}{2}\bar{\theta}\bar{\sigma}^\nu\sigma^\mu\bar{\varepsilon}\cdot\theta\theta}\frac{\partial V_\nu}{\partial x^\mu} \\
&\quad + \left[ (\varepsilon\sigma^\mu)_\alpha\frac{\partial m(x)}{\partial x^\mu} + \bar{\varepsilon}_\alpha d(x) \right] \bar{\theta}^\alpha\cdot\theta\theta + \left[ -(\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial m^\dagger(x)}{\partial x^\mu} + \varepsilon_\alpha d(x) \right] \theta^\alpha\cdot\bar{\theta}\bar{\theta} \\
&= -i\frac{1}{2}\varepsilon\sigma^\mu\bar{\sigma}^\nu\theta\cdot\bar{\theta}\bar{\theta}\frac{\partial V_\nu}{\partial x^\mu} + i\frac{1}{2}\bar{\theta}\bar{\sigma}^\nu\sigma^\mu\bar{\varepsilon}\cdot\theta\theta\frac{\partial V_\nu}{\partial x^\mu} \\
&\quad + \varepsilon\sigma^\mu\bar{\theta}\cdot\theta\theta\frac{\partial m(x)}{\partial x^\mu} + \bar{\varepsilon}\bar{\theta}\cdot\theta\theta d(x) + \theta\sigma^\mu\bar{\varepsilon}\cdot\bar{\theta}\theta\frac{\partial m^\dagger(x)}{\partial x^\mu} - \varepsilon\theta\cdot\bar{\theta}\theta d(x) \\
&\Leftarrow \varepsilon\sigma^\mu\bar{\sigma}^\nu\theta = \theta\sigma^\nu\bar{\sigma}^\mu\varepsilon, \bar{\theta}\bar{\sigma}^\nu\sigma^\mu\bar{\varepsilon} = \bar{\varepsilon}\bar{\sigma}^\mu\sigma^\nu\bar{\theta} \\
&= -i\frac{1}{2}\theta\sigma^\nu\bar{\sigma}^\mu\varepsilon\cdot\bar{\theta}\bar{\theta}\frac{\partial V_\nu}{\partial x^\mu} + i\frac{1}{2}\bar{\varepsilon}\bar{\sigma}^\mu\sigma^\nu\bar{\theta}\cdot\theta\theta\frac{\partial V_\nu}{\partial x^\mu} \\
&\quad + \varepsilon\sigma^\mu\bar{\theta}\cdot\theta\theta\frac{\partial m(x)}{\partial x^\mu} + \bar{\varepsilon}\bar{\theta}\cdot\theta\theta d(x) + \theta\sigma^\mu\bar{\varepsilon}\cdot\bar{\theta}\theta\frac{\partial m^\dagger(x)}{\partial x^\mu} - \varepsilon\theta\cdot\bar{\theta}\theta d(x) \\
&= \left[ i\frac{1}{2}(\bar{\varepsilon}\bar{\sigma}^\mu\sigma^\nu)_\alpha\frac{\partial V_\nu}{\partial x^\mu} + (\varepsilon\sigma^\mu)_\alpha\frac{\partial m(x)}{\partial x^\mu} + \bar{\varepsilon}_\alpha d(x) \right] \bar{\theta}^\alpha\cdot\theta\theta \\
&\quad + \theta^\alpha \left[ -i\frac{1}{2}(\sigma^\nu\bar{\sigma}^\mu\varepsilon)_\alpha\frac{\partial V_\nu}{\partial x^\mu} + (\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial m^\dagger(x)}{\partial x^\mu} - \varepsilon_\alpha d(x) \right] \cdot\bar{\theta}\bar{\theta} \\
\Rightarrow \delta\bar{\psi}_\alpha(x) &= \frac{1}{2}(\bar{\varepsilon}\bar{\sigma}^\mu\sigma^\nu)_\alpha\frac{\partial V_\nu}{\partial x^\mu} - i(\varepsilon\sigma^\mu)_\alpha\frac{\partial m(x)}{\partial x^\mu} - i\bar{\varepsilon}_\alpha d(x) \\
\delta\psi_\alpha(x) &= \frac{1}{2}(\sigma^\nu\bar{\sigma}^\mu\varepsilon)_\alpha\frac{\partial V_\nu}{\partial x^\mu} + i(\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial m^\dagger(x)}{\partial x^\mu} - i\varepsilon_\alpha d(x)
\end{aligned}$$

(\*)  $\theta\theta\bar{\theta}\bar{\theta}$ 

$$\begin{aligned}
\frac{1}{2}\delta d(x)\theta\theta\bar{\theta}\bar{\theta} &= -\left(i\varepsilon\sigma^\mu\bar{\theta}\frac{\partial i\theta\theta\bar{\theta}\bar{\psi}(x)}{\partial x^\mu}-i\theta\sigma^\mu\bar{\varepsilon}\frac{\partial(-i)\bar{\theta}\bar{\theta}\theta\psi(x)}{\partial x^\mu}\right) \\
&= (\varepsilon\sigma^\mu)_\beta\bar{\theta}^\beta\cdot\theta\theta\bar{\theta}_\alpha\frac{\partial\bar{\psi}^\alpha(x)}{\partial x^\mu}+\theta^\alpha(\sigma^\mu\bar{\varepsilon})_\alpha\cdot\bar{\theta}\bar{\theta}\frac{\partial\psi^\beta(x)}{\partial x^\mu}\theta_\beta \\
&\Leftarrow\theta^\alpha\theta_\beta=\frac{1}{2}\delta_\beta^\alpha\theta\theta,\bar{\theta}_\alpha\bar{\theta}^\beta=\frac{1}{2}\delta_\alpha^\beta\bar{\theta}\bar{\theta} \\
&= (\varepsilon\sigma^\mu)_\beta\left(-\frac{1}{2}\delta_\alpha^\beta\bar{\theta}\bar{\theta}\right)\cdot\theta\theta\frac{\partial\bar{\psi}^\alpha(x)}{\partial x^\mu}+\frac{1}{2}\delta_\beta^\alpha\theta\theta(\sigma^\mu\bar{\varepsilon})_\alpha\cdot\bar{\theta}\bar{\theta}\frac{\partial\psi^\beta(x)}{\partial x^\mu} \\
&= -\left(\varepsilon\sigma^\mu\frac{\partial\bar{\psi}(x)}{\partial x^\mu}\right)\frac{1}{2}\bar{\theta}\bar{\theta}\cdot\theta\theta-\frac{1}{2}\left(\frac{\partial\psi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon}\right)\theta\theta\cdot\bar{\theta}\bar{\theta} \\
\Rightarrow\delta d(x) &= -\left(\varepsilon\sigma^\mu\frac{\partial\bar{\psi}(x)}{\partial x^\mu}+\frac{\partial\psi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon}\right)=-\frac{\partial}{\partial x^\mu}\left(\varepsilon\sigma^\mu\bar{\psi}(x)+\psi(x)\sigma^\mu\bar{\varepsilon}\right)
\end{aligned}$$

以上から、

$$\begin{aligned}
\delta C(x) &= -i(\varepsilon\chi(x)-\bar{\varepsilon}\bar{\chi}(x)) \\
\delta\chi_\alpha(x) &= -\left(2\varepsilon_\alpha m(x)-i(\sigma^\mu\bar{\varepsilon})_\alpha V_\mu-(\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial C(x)}{\partial x^\mu}\right) \\
\delta\bar{\chi}_\alpha(x) &= -\left(2\bar{\varepsilon}_\alpha m^\dagger(x)+i(\varepsilon\sigma^\mu)_\alpha V_\mu-(\varepsilon\sigma^\mu)_\alpha\frac{\partial C(x)}{\partial x^\mu}\right) \\
\delta m(x) &= -\bar{\varepsilon}\bar{\psi}(x)-i\frac{1}{2}\frac{\partial\chi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon} \\
\delta m^\dagger(x) &= -\varepsilon\psi(x)+i\frac{1}{2}\varepsilon\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu} \\
\delta V_\mu &= -i(\varepsilon\sigma^\mu\bar{\psi}+\bar{\varepsilon}\sigma^\mu\psi)-\frac{1}{2}\left(\varepsilon\sigma^\nu\bar{\sigma}^\mu\frac{\partial\chi(x)}{\partial x^\nu}+\frac{\partial\bar{\chi}(x)}{\partial x^\nu}\bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon}\right) \\
\delta\psi_\alpha(x) &= \frac{1}{2}(\sigma^\nu\bar{\sigma}^\mu\varepsilon)_\alpha\frac{\partial V_\nu}{\partial x^\mu}+i(\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial m^\dagger(x)}{\partial x^\mu}-i\varepsilon_\alpha d(x) \\
\delta\bar{\psi}_\alpha(x) &= \frac{1}{2}(\bar{\varepsilon}\sigma^\mu\sigma^\nu)_\alpha\frac{\partial V_\nu}{\partial x^\mu}-i(\varepsilon\sigma^\mu)_\alpha\frac{\partial m(x)}{\partial x^\mu}-i\bar{\varepsilon}_\alpha d(x) \\
\delta d(x) &= -\left(\varepsilon\sigma^\mu\frac{\partial\bar{\psi}(x)}{\partial x^\mu}+\frac{\partial\psi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon}\right)
\end{aligned}$$

を得る。ところで、

$$\delta\psi_\alpha(x)=\frac{1}{2}(\sigma^\nu\bar{\sigma}^\mu\varepsilon)_\alpha\frac{\partial V_\nu}{\partial x^\mu}+i(\sigma^\mu\bar{\varepsilon})_\alpha\frac{\partial m^\dagger(x)}{\partial x^\mu}-i\varepsilon_\alpha d(x)$$

には、

$$\delta\bar{\chi}_\alpha(x)=-\left(2\bar{\varepsilon}_\alpha m^\dagger(x)+i(\varepsilon\sigma^\mu)_\alpha V_\mu-(\varepsilon\sigma^\mu)_\alpha\frac{\partial C(x)}{\partial x^\mu}\right)$$

が含まれるので、



$$\begin{aligned}
\delta\theta\psi(x) &= \frac{1}{2}(\theta\sigma^\nu\bar{\sigma}^\mu\varepsilon)\frac{\partial V_\nu}{\partial x^\mu} + i(\theta\sigma^\mu\bar{\varepsilon})\frac{\partial m^\dagger(x)}{\partial x^\mu} - i\theta\varepsilon d(x) \\
&\Leftarrow \theta\sigma^\nu\bar{\sigma}^\mu\varepsilon = \varepsilon\sigma^\mu\bar{\sigma}^\nu\theta, \theta\sigma^\mu\bar{\varepsilon} = -\bar{\varepsilon}\bar{\sigma}^\mu\theta \\
&= \frac{1}{2}(\varepsilon\sigma^\mu\bar{\sigma}^\nu\theta)\frac{\partial V_\nu}{\partial x^\mu} - i(\bar{\varepsilon}\bar{\sigma}^\mu\theta)\frac{\partial m^\dagger(x)}{\partial x^\mu} - i\theta\varepsilon d(x) \\
&\Leftarrow \delta\bar{\chi}_\alpha(x) = -\left(2\bar{\varepsilon}_\alpha m^\dagger(x) + i(\varepsilon\sigma^\nu)_\alpha V_\nu - (\varepsilon\sigma^\nu)_\alpha \frac{\partial C(x)}{\partial x^\nu}\right) \\
&\quad \frac{1}{2}i\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu}\bar{\sigma}^\mu\theta = -i\bar{\varepsilon}\bar{\sigma}^\mu\theta\frac{\partial m^\dagger(x)}{\partial x^\mu} + \frac{1}{2}(\varepsilon\sigma^\nu\bar{\sigma}^\mu\theta)\frac{\partial V_\nu}{\partial x^\mu} + i\frac{1}{2}(\varepsilon\sigma^\nu\bar{\sigma}^\mu\theta)\frac{\partial^2 C(x)}{\partial x^\mu\partial x^\nu} \\
&= \frac{1}{2}(\varepsilon\sigma^\mu\bar{\sigma}^\nu\theta)\frac{\partial V_\nu}{\partial x^\mu} + \left[\frac{1}{2}i\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu}\bar{\sigma}^\mu\theta - \frac{1}{2}(\varepsilon\sigma^\nu\bar{\sigma}^\mu\theta)\frac{\partial V_\nu}{\partial x^\mu} - i\frac{1}{2}(\varepsilon\sigma^\nu\bar{\sigma}^\mu\theta)\frac{\partial^2 C(x)}{\partial x^\mu\partial x^\nu}\right] - i\theta\varepsilon d(x) \\
&\Leftarrow \theta\sigma^\nu\bar{\sigma}^\mu\varepsilon = \varepsilon\sigma^\mu\bar{\sigma}^\nu\theta, \theta\sigma^\mu\bar{\varepsilon} = -\bar{\varepsilon}\bar{\sigma}^\mu\theta \\
&= \frac{1}{2}(\varepsilon[\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu]\theta)\frac{\partial V_\nu}{\partial x^\mu} + \frac{1}{2}i\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu}\bar{\sigma}^\mu\theta - i\frac{1}{2}(\varepsilon\sigma^\nu\bar{\sigma}^\mu\theta)\frac{\partial^2 C(x)}{\partial x^\mu\partial x^\nu} - i\theta\varepsilon d(x) \\
&\Leftarrow \sigma^\mu\bar{\sigma}^\rho = g^{\mu\rho} + i\sigma^{\mu\rho}, \bar{\sigma}^\mu\sigma^\rho = g^{\mu\rho} + i\bar{\sigma}^{\mu\rho} \\
&= \frac{1}{2}(\varepsilon 2i\sigma^{\mu\nu}\theta)\frac{\partial V_\nu}{\partial x^\mu} + \frac{1}{2}i\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu}\bar{\sigma}^\mu\theta - i\frac{1}{2}(\varepsilon g^{\nu\mu}\theta)\frac{\partial^2 C(x)}{\partial x^\mu\partial x^\nu} - i\theta\varepsilon d(x) \\
&\Leftarrow F_{\mu\nu} = \frac{\partial V_\nu}{\partial x^\mu} - \frac{\partial V_\mu}{\partial x_\nu} \\
&= \frac{1}{2}(\varepsilon i\sigma^{\mu\nu}\theta)F_{\mu\nu} - \frac{1}{2}i\theta\sigma^\mu\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu} - i\frac{1}{2}\varepsilon\theta\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu} - i\theta\varepsilon d(x) \\
&= \frac{1}{2}(\varepsilon i\sigma^{\mu\nu}\theta)F_{\mu\nu} - \frac{1}{2}i\theta\sigma^\mu\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu} - i\theta\varepsilon\left(d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right)
\end{aligned}$$

以上から、

$$\begin{aligned}
\delta\theta\psi(x) &= \frac{1}{2}(\varepsilon i\sigma^{\mu\nu}\theta)V_{\mu\nu} - \frac{1}{2}i\theta\sigma^\mu\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu} - i\theta\varepsilon\left(d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right) \\
&= -\frac{1}{2}i\theta\sigma^{\mu\nu}\varepsilon V_{\mu\nu} - \frac{1}{2}i\theta\sigma^\mu\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu} - i\theta\varepsilon\left(d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right) \\
&\Rightarrow \delta\theta\psi(x) + \frac{1}{2}i\theta\sigma^\mu\frac{\partial\delta\bar{\chi}(x)}{\partial x^\mu} = -\frac{1}{2}i\theta\sigma^{\mu\nu}\varepsilon V_{\mu\nu} - i\theta\varepsilon\left(d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right) \\
&\quad \theta\delta\left(\psi(x) + \frac{1}{2}i\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu}\right) = -\frac{1}{2}i\theta\sigma^{\mu\nu}\varepsilon V_{\mu\nu} - i\theta\varepsilon\left(d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right) \\
&\Rightarrow \delta\left(\psi(x) + \frac{1}{2}i\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu}\right)_\alpha = -\frac{1}{2}i(\sigma^{\mu\nu}\varepsilon)_\alpha V_{\mu\nu} - i\varepsilon_\alpha\left(d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right)
\end{aligned}$$

がわかる。そこで、 $\psi(x)$ と $d(x)$ をあらためて次のように定義する：

$$\delta\lambda_\alpha(x) = -\frac{1}{2}i(\sigma^{\mu\nu}\varepsilon)_\alpha V_{\mu\nu} - i\varepsilon_\alpha D(x) \quad \left\{ \begin{array}{l} \lambda_\alpha(x) \equiv \psi_\alpha(x) + \frac{1}{2}i\left(\sigma^\mu \frac{\partial\bar{\chi}(x)}{\partial x^\mu}\right)_\alpha \\ D(x) \equiv d(x) + \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu} \end{array} \right.$$

その結果、

$$\begin{aligned} \delta m(x) &= -\bar{\varepsilon}\bar{\psi}(x) - i\frac{1}{2}\frac{\partial\chi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon} = -\left(\bar{\lambda}(x) + \frac{1}{2}\frac{\partial\chi(x)}{\partial x^\mu}i\sigma^\mu\right)\bar{\varepsilon} - i\frac{1}{2}\frac{\partial\chi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon} \\ &= -\bar{\lambda}(x)\bar{\varepsilon} - i\frac{\partial\chi(x)}{\partial x^\mu}\sigma^\mu\bar{\varepsilon} \end{aligned}$$

$$\begin{aligned} \delta m^\dagger(x) &= -\varepsilon\psi(x) + i\frac{1}{2}\varepsilon\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu} = -\varepsilon\left(\lambda(x) - \frac{1}{2}i\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu}\right) + i\frac{1}{2}\varepsilon\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu} \\ &= -\varepsilon\lambda(x) + i\varepsilon\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu} \end{aligned}$$

$$\begin{aligned} \delta V^\mu &= -i\left(\varepsilon\sigma^\mu\bar{\psi} + \bar{\varepsilon}\bar{\sigma}^\mu\psi\right) - \frac{1}{2}\left(\varepsilon\sigma^\nu\bar{\sigma}^\mu\frac{\partial\chi(x)}{\partial x^\nu} + \frac{\partial\bar{\chi}(x)}{\partial x^\nu}\bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon}\right) \\ &= -i\left(-\left(\bar{\lambda}(x) + \frac{1}{2}i\frac{\partial\chi(x)}{\partial x^\nu}\sigma^\nu\right)\bar{\sigma}^\mu\varepsilon + \bar{\varepsilon}\bar{\sigma}^\mu\left(\lambda(x) - \frac{1}{2}i\sigma^\nu\frac{\partial\bar{\chi}(x)}{\partial x^\nu}\right)\right) \\ &\quad - \frac{1}{2}\left(\varepsilon\sigma^\nu\bar{\sigma}^\mu\frac{\partial\chi(x)}{\partial x^\nu} + \frac{\partial\bar{\chi}(x)}{\partial x^\nu}\bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon}\right) \\ &= i\bar{\lambda}(x)\bar{\sigma}^\mu\varepsilon - \frac{1}{2}\frac{\partial\chi(x)}{\partial x^\nu}\sigma^\nu\bar{\sigma}^\mu\varepsilon - i\bar{\varepsilon}\bar{\sigma}^\mu\lambda(x) - \frac{1}{2}\bar{\varepsilon}\bar{\sigma}^\mu\sigma^\nu\frac{\partial\bar{\chi}(x)}{\partial x^\nu} \\ &\quad - \frac{1}{2}\varepsilon\sigma^\nu\bar{\sigma}^\mu\frac{\partial\chi(x)}{\partial x^\nu} - \frac{1}{2}\frac{\partial\bar{\chi}(x)}{\partial x^\nu}\bar{\sigma}^\mu\sigma^\nu\bar{\varepsilon} \\ &\Leftarrow \theta\sigma^\nu\bar{\sigma}^\mu\varepsilon = \varepsilon\sigma^\mu\bar{\sigma}^\nu\theta, \theta\sigma^\mu\bar{\varepsilon} = -\bar{\varepsilon}\bar{\sigma}^\mu\theta \\ &= -i\varepsilon\sigma^\mu\bar{\lambda}(x) - \frac{1}{2}\varepsilon(\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu)\frac{\partial\chi(x)}{\partial x^\nu} + i\lambda(x)\sigma^\mu\bar{\varepsilon} - \frac{1}{2}\bar{\varepsilon}(\bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu)\frac{\partial\bar{\chi}(x)}{\partial x^\nu} \\ &= -i\varepsilon\sigma^\mu\bar{\lambda}(x) - \frac{1}{2}\varepsilon 2g^{\mu\nu}\frac{\partial\chi(x)}{\partial x^\nu} + i\lambda(x)\sigma^\mu\bar{\varepsilon} - \frac{1}{2}\bar{\varepsilon} 2g^{\mu\nu}\frac{\partial\bar{\chi}(x)}{\partial x^\nu} \\ &= -i\varepsilon\sigma^\mu\bar{\lambda}(x) - \frac{1}{2}\varepsilon 2\frac{\partial\chi(x)}{\partial x_\mu} + i\lambda(x)\sigma^\mu\bar{\varepsilon} - \frac{1}{2}\bar{\varepsilon} 2\frac{\partial\bar{\chi}(x)}{\partial x_\mu} \\ &= -i\left(\varepsilon\sigma^\mu\bar{\lambda}(x) - \lambda(x)\sigma^\mu\bar{\varepsilon}\right) - \frac{\partial}{\partial x_\mu}\left(\varepsilon\chi(x) + \bar{\varepsilon}\bar{\chi}(x)\right) \end{aligned}$$

$$\begin{aligned} \delta d(x) &= \delta\left(D(x) - \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu}\right) = \delta D(x) - \frac{1}{2}\frac{\partial^2 \delta C(x)}{\partial x^\mu \partial x_\mu} = \delta D(x) - \frac{1}{2}\frac{\partial^2(-i)(\varepsilon\chi(x) - \bar{\varepsilon}\bar{\chi}(x))}{\partial x^\mu \partial x_\mu} \\ &\Leftarrow \frac{\partial}{\partial x^\nu}\bar{\varepsilon}\bar{\sigma}^\nu\psi(x) = \frac{\partial}{\partial x^\nu}\left(\bar{\varepsilon}\bar{\sigma}^\nu\lambda(x) - \frac{1}{2}i\bar{\varepsilon}\bar{\sigma}^\nu\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu}\right) = \bar{\varepsilon}\bar{\sigma}^\nu\frac{\partial\lambda(x)}{\partial x^\nu} - \frac{1}{2}i\bar{\varepsilon}g^{\nu\mu}\frac{\partial^2\bar{\chi}(x)}{\partial x^\nu \partial x^\mu} \end{aligned}$$

$$\begin{aligned}
&= \overline{\varepsilon\sigma^\nu} \frac{\partial\lambda(x)}{\partial x^\nu} - \frac{1}{2} i\overline{\varepsilon} \frac{\partial^2\bar{\chi}(x)}{\partial x^\mu\partial x_\mu} \\
&= \delta D(x) + \frac{1}{2} i\varepsilon \frac{\partial^2\chi(x)}{\partial x^\mu\partial x_\mu} - \frac{1}{2} i\overline{\varepsilon} \frac{\partial^2\bar{\chi}(x)}{\partial x^\mu\partial x_\mu} = \delta D(x) + \left( \frac{\partial}{\partial x^\nu} \overline{\varepsilon\sigma^\nu} \psi(x) - \overline{\varepsilon\sigma^\nu} \frac{\partial\lambda(x)}{\partial x^\nu} + \text{h.c.} \right) \\
&= - \left( \varepsilon\sigma^\mu \frac{\partial\bar{\psi}(x)}{\partial x^\mu} + \frac{\partial\psi(x)}{\partial x^\mu} \sigma^\mu \overline{\varepsilon} \right) \\
&\Rightarrow \delta D(x) + \left( \overline{\varepsilon\sigma^\nu} \frac{\partial\psi(x)}{\partial x^\nu} - \overline{\varepsilon\sigma^\nu} \frac{\partial\lambda(x)}{\partial x^\nu} + \text{h.c.} \right) = - \left( \varepsilon\sigma^\mu \frac{\partial\bar{\psi}(x)}{\partial x^\mu} + \frac{\partial\psi(x)}{\partial x^\mu} \sigma^\mu \overline{\varepsilon} \right) \\
&\Rightarrow \delta D(x) = - \left( \varepsilon\sigma^\mu \frac{\partial\bar{\psi}(x)}{\partial x^\mu} + \frac{\partial\psi(x)}{\partial x^\mu} \sigma^\mu \overline{\varepsilon} \right) - \left( - \frac{\partial\psi(x)}{\partial x^\nu} \sigma^\nu \overline{\varepsilon} + \frac{\partial\lambda(x)}{\partial x^\nu} \sigma^\nu \overline{\varepsilon} + \text{h.c.} \right) \\
&= - \left( \frac{\partial\lambda(x)}{\partial x^\nu} \sigma^\nu \overline{\varepsilon} + \text{h.c.} \right) = - \left( \frac{\partial\lambda(x)}{\partial x^\nu} \sigma^\nu \overline{\varepsilon} + \varepsilon\sigma^\nu \frac{\partial\bar{\lambda}(x)}{\partial x^\nu} \right) = - \frac{\partial}{\partial x^\nu} (\lambda(x) \sigma^\nu \overline{\varepsilon} + \varepsilon\sigma^\nu \bar{\lambda}(x))
\end{aligned}$$

以上から、

$$\begin{aligned}
V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta m(x) - i\bar{\theta}\bar{\theta} m^\dagger(x) + \theta\sigma^\mu\bar{\theta}V_\mu \\
&\quad + i\theta\theta\bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial\chi(x)}{\partial x^\mu} \right) - i\bar{\theta}\bar{\theta}\theta \left( \lambda(x) - \frac{1}{2} i\sigma^\mu \frac{\partial\bar{\chi}(x)}{\partial x^\mu} \right) + \frac{1}{2} \theta\theta\bar{\theta}\bar{\theta} \left( D(x) - \frac{1}{2} \frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu} \right)
\end{aligned}$$

$$\delta C(x) = -i(\varepsilon\chi(x) - \bar{\varepsilon}\bar{\chi}(x))$$

$$\delta\chi_\alpha(x) = - \left( 2\varepsilon_\alpha m(x) - i(\sigma^\mu\bar{\varepsilon})_\alpha \left( V_\mu - i \frac{\partial C(x)}{\partial x^\mu} \right) \right), \quad \delta\bar{\chi}_\alpha(x) = - \left( 2\bar{\varepsilon}_\alpha m^\dagger(x) + i(\varepsilon\sigma^\mu)_\alpha \left( V_\mu + i \frac{\partial C(x)}{\partial x^\mu} \right) \right)$$

$$\delta m(x) = -\bar{\varepsilon}\bar{\lambda}(x) - i \frac{\partial\chi(x)}{\partial x^\mu} \sigma^\mu \overline{\varepsilon}, \quad \delta m^\dagger(x) = -\varepsilon\lambda(x) + i\varepsilon\sigma^\mu \frac{\partial\bar{\chi}(x)}{\partial x^\mu}$$

$$\delta V_\mu = -i(\varepsilon\sigma_\mu\bar{\lambda}(x) - \lambda(x)\sigma_\mu\overline{\varepsilon}) - \frac{\partial}{\partial x^\mu} (\varepsilon\chi(x) + \bar{\varepsilon}\bar{\chi}(x))$$

$$\delta\lambda_\alpha(x) = -\frac{1}{2} i(\sigma^{\mu\nu}\varepsilon)_\alpha F_{\mu\nu} - i\varepsilon_\alpha D(x), \quad \delta\bar{\lambda}_\alpha(x) = -\frac{1}{2} i(\bar{\varepsilon}\sigma^{\mu\nu})_\alpha F_{\mu\nu} + i\bar{\varepsilon}_\alpha D(x)$$

$$\delta D(x) = -\frac{\partial}{\partial x^\nu} (\lambda(x) \sigma^\nu \overline{\varepsilon} + \varepsilon\sigma^\nu \bar{\lambda}(x))$$

## カイラルゲージ超場

ここで、場の強さ： $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$  の変換性を調べると

$$\begin{aligned}
\delta F_{\mu\nu} &= \partial_\mu \delta V_\nu - \partial_\nu \delta V_\mu = \partial_\mu \left( -i(\varepsilon\sigma_\nu\bar{\lambda}(x) - \lambda(x)\sigma_\nu\overline{\varepsilon}) - \frac{\partial}{\partial x^\nu} (\varepsilon\chi(x) + \bar{\varepsilon}\bar{\chi}(x)) \right) \\
&\quad - \partial_\nu \left( -i(\varepsilon\sigma_\mu\bar{\lambda}(x) - \lambda(x)\sigma_\mu\overline{\varepsilon}) - \frac{\partial}{\partial x^\mu} (\varepsilon\chi(x) + \bar{\varepsilon}\bar{\chi}(x)) \right) \\
&= -i(\varepsilon\sigma_\nu\partial_\mu\bar{\lambda}(x) - \partial_\mu\lambda(x)\sigma_\nu\overline{\varepsilon}) + i(\varepsilon\sigma_\mu\partial_\nu\bar{\lambda}(x) - \partial_\nu\lambda(x)\sigma_\mu\overline{\varepsilon})
\end{aligned}$$

になる。つまり超対称性変換は、 $(F_{\mu\nu}, \lambda, D)$  で閉じていることがわかる。

$$\begin{aligned}\delta V_{\mu\nu} &= -i(\varepsilon\sigma_\nu\partial_\mu\bar{\lambda}(x) - \partial_\mu\lambda(x)\sigma_\nu\bar{\varepsilon}) + i(\varepsilon\sigma_\mu\partial_\nu\bar{\lambda}(x) - \partial_\nu\lambda(x)\sigma_\mu\bar{\varepsilon}) \\ \delta\lambda_\alpha(x) &= -\frac{1}{2}i(\sigma^{\mu\nu}\varepsilon)_\alpha F_{\mu\nu} - i\varepsilon_\alpha D(x), \quad \delta\bar{\lambda}_\alpha(x) = -\frac{1}{2}i(\bar{\varepsilon}\sigma^{\mu\nu})_\alpha F_{\mu\nu} + i\bar{\varepsilon}_\alpha D(x) \\ \delta D(x) &= -\frac{\partial}{\partial x^\nu}(\lambda(x)\sigma^\nu\bar{\varepsilon} + \varepsilon\sigma^\nu\bar{\lambda}(x))\end{aligned}$$

これらは、スピノールカイラルゲージ超場  $W_\alpha$  の成分になっていることが知られており、

$$W_\alpha = \bar{D}^2 D_\alpha V = \bar{D}_\beta \bar{D}^\beta D_\alpha V \quad \text{with} \quad \bar{D}_\beta W_\alpha = 0$$

で構築することが可能である。ここで

$$\begin{aligned}V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta m(x) - i\bar{\theta}\bar{\theta} m^\dagger(x) + \theta\sigma^\mu\bar{\theta}V_\mu \\ &\quad + i\theta\bar{\theta}\bar{\theta}\left(\bar{\lambda}(x) - \frac{1}{2}i\bar{\sigma}^\mu\frac{\partial\chi(x)}{\partial x^\mu}\right) - i\bar{\theta}\theta\theta\left(\lambda(x) - \frac{1}{2}i\sigma^\mu\frac{\partial\bar{\chi}(x)}{\partial x^\mu}\right) + \frac{1}{2}\theta\bar{\theta}\bar{\theta}\bar{\theta}\left(D(x) - \frac{1}{2}\frac{\partial^2 C(x)}{\partial x^\mu\partial x_\mu}\right) \\ D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\bar{\theta})_\alpha\frac{\partial}{\partial x^\mu}, \quad \bar{D}_\alpha = -\frac{\partial}{\partial\bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial x^\mu}\end{aligned}$$

なので、

$$\begin{aligned}\bar{\varepsilon}^\beta\bar{D}_\beta &= -\bar{\varepsilon}_\beta\bar{D}^\beta = -\bar{\varepsilon}^\beta\frac{\partial}{\partial\bar{\theta}^\beta} - i(\theta\sigma^\mu\bar{\varepsilon})_\beta\frac{\partial}{\partial x^\mu} = -\bar{\varepsilon}_\beta\frac{\partial}{\partial\bar{\theta}^\beta} + i(\bar{\varepsilon}\sigma^\mu\theta)\frac{\partial}{\partial x^\mu} \\ \Rightarrow \bar{D}^\beta &= \frac{\partial}{\partial\bar{\theta}^\beta} - i(\bar{\sigma}^\mu\theta)^\beta\frac{\partial}{\partial x^\mu}\end{aligned}$$

を用いて、

$$\begin{aligned}\bar{D}_\alpha\bar{D}^\alpha &= \left(-\frac{\partial}{\partial\bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial x^\mu}\right)\left(\frac{\partial}{\partial\bar{\theta}^\alpha} - i(\bar{\sigma}^\nu\theta)^\alpha\frac{\partial}{\partial x^\nu}\right) \\ &= -\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial\bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial x^\mu} + i\frac{\partial}{\partial\bar{\theta}^\alpha}(\bar{\sigma}^\nu\theta)^\alpha\frac{\partial}{\partial x^\nu} + (\theta\sigma^\mu)_\alpha(\bar{\sigma}^\nu\theta)^\alpha\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} \\ &\Leftarrow (\theta\sigma^\mu)_\alpha(\bar{\sigma}^\nu\theta)^\alpha = \theta^{\alpha'}(\sigma^\mu)_{\alpha'\alpha}(\bar{\sigma}^\nu)^{\alpha\beta'}\theta_{\beta'} = \frac{1}{2}\delta_{\beta'}^{\alpha'}\theta\theta(\sigma^\mu)_{\alpha'\alpha}(\bar{\sigma}^\nu)^{\alpha\beta'} = \frac{1}{2}\text{Tr}(\bar{\sigma}^\nu\sigma^\mu)\theta\theta = g^{\mu\nu}\theta\theta \\ &= -\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial\bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial x^\mu} + i\frac{\partial}{\partial\bar{\theta}^\alpha}(\bar{\sigma}^\nu\theta)^\alpha\frac{\partial}{\partial x^\nu} + (g^{\mu\nu}\theta\theta)\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} \\ &= -\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial\bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\frac{\partial}{\partial x^\mu} + i\frac{\partial}{\partial\bar{\theta}^\alpha}(\bar{\sigma}^\nu\theta)^\alpha\frac{\partial}{\partial x^\nu} + \theta\theta\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x_\mu}\end{aligned}$$

である。更に、

$$\begin{aligned}i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\bar{\varepsilon}\bar{\theta} &= i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\bar{\theta}_\beta\bar{\varepsilon}^\beta = i(\theta\sigma^\mu\bar{\varepsilon}) \\ i\frac{\partial}{\partial\bar{\theta}^\alpha}(\bar{\sigma}^\nu\theta)^\alpha\bar{\varepsilon}\bar{\theta} &= i\frac{\partial}{\partial\bar{\theta}^\alpha}\bar{\varepsilon}\bar{\theta}(\bar{\sigma}^\nu\theta)^\alpha = -i\frac{\partial}{\partial\bar{\theta}^\alpha}\bar{\theta}^\beta\bar{\varepsilon}_\beta(\bar{\sigma}^\nu\theta)^\alpha = -i(\bar{\varepsilon}\bar{\sigma}^\nu\theta) = i(\theta\sigma^\mu\bar{\varepsilon}) \\ i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\bar{\theta}\bar{\theta} &= i(\theta\sigma^\mu)_\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}\bar{\theta}_\beta\bar{\theta}^\beta = 2i(\theta\sigma^\mu\bar{\theta})\end{aligned}$$

$$i \frac{\partial}{\partial \theta^\alpha} (\bar{\sigma}^\nu \theta)^\alpha \bar{\theta} \theta = i \frac{\partial}{\partial \theta^\alpha} \bar{\theta} \theta (\bar{\sigma}^\nu \theta)^\alpha = -i \frac{\partial}{\partial \theta^\alpha} \bar{\theta}^\beta \bar{\theta}_\beta (\bar{\sigma}^\nu \theta)^\alpha = -2i (\bar{\theta} \bar{\sigma}^\nu \theta) = 2i (\theta \sigma^\mu \bar{\theta})$$

より、

$$\begin{aligned} \bar{D}_\alpha \bar{D}^\alpha &= -\frac{\partial}{\partial \theta^\alpha} \frac{\partial}{\partial \bar{\theta}_\alpha} + i(\theta \sigma^\mu)_\alpha \frac{\partial}{\partial \theta_\alpha} \frac{\partial}{\partial x^\mu} + i \frac{\partial}{\partial \theta^\alpha} (\bar{\sigma}^\nu \theta)^\alpha \frac{\partial}{\partial x^\nu} + \theta \theta \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} \\ &= -\frac{\partial}{\partial \theta^\alpha} \frac{\partial}{\partial \bar{\theta}_\alpha} + 2i(\theta \sigma^\nu)_\alpha \frac{\partial}{\partial \theta_\alpha} \frac{\partial}{\partial x^\nu} + \theta \theta \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} \\ &= -\frac{\partial}{\partial \theta^\alpha} \frac{\partial}{\partial \bar{\theta}_\alpha} + 2i \frac{\partial}{\partial \theta^\alpha} (\bar{\sigma}^\nu \theta)^\alpha \frac{\partial}{\partial x^\nu} + \theta \theta \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} \end{aligned}$$

$$\begin{aligned} D_\alpha V &= \left( \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} \right) V \\ &= \frac{\partial}{\partial \theta^\alpha} \left( \begin{aligned} &i\theta \chi(x) + i\theta \theta m(x) + \theta \sigma^\mu \bar{\theta} V_\mu \\ &+ i\theta \theta \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i \bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) - i\bar{\theta} \theta \left( \lambda(x) - \frac{1}{2} i \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right) + \frac{1}{2} \theta \theta \bar{\theta} \theta \left( D(x) - \frac{1}{2} \frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu} \right) \\ &- i(\sigma^\nu \bar{\theta})_\alpha \frac{\partial}{\partial x^\nu} \left( C(x) + i\theta \chi(x) - i\bar{\theta} \bar{\chi}(x) + i\theta \theta m(x) + \theta \sigma^\mu \bar{\theta} V_\mu + i\theta \theta \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i \bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) \right) \end{aligned} \right) \\ &= i\chi_\alpha(x) + 2i\theta_\alpha m(x) + (\sigma^\mu \bar{\theta})_\alpha V_\mu + 2i\theta_\alpha \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i \bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) - i\bar{\theta} \theta \left( \lambda(x) - \frac{1}{2} i \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right)_\alpha \\ &\quad + \frac{1}{2} 2\theta_\alpha \bar{\theta} \theta \left( D(x) - \frac{1}{2} \frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu} \right) \\ &\quad - i(\sigma^\nu \bar{\theta})_\alpha \frac{\partial}{\partial x^\nu} \left( C(x) + i\theta \chi(x) - i\bar{\theta} \bar{\chi}(x) + i\theta \theta m(x) + \theta \sigma^\mu \bar{\theta} V_\mu + i\theta \theta \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i \bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) \right) \\ \bar{D}_\beta \bar{D}^\beta D_\alpha V &= \left( -\frac{\partial}{\partial \bar{\theta}^\beta} \frac{\partial}{\partial \bar{\theta}_\beta} + 2i(\theta \sigma^\mu)_\beta \frac{\partial}{\partial \bar{\theta}_\beta} \frac{\partial}{\partial x^\mu} - \theta \theta \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} \right) D_\alpha V \\ &= \left( -\frac{\partial}{\partial \bar{\theta}^\beta} \frac{\partial}{\partial \bar{\theta}_\beta} \right) \left( \begin{aligned} &-i\bar{\theta} \theta \left( \lambda(x) - \frac{1}{2} i \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right)_\alpha + \frac{1}{2} 2\theta_\alpha \bar{\theta} \theta \left( D(x) - \frac{1}{2} \frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu} \right) \\ &-i(\sigma^\nu \bar{\theta})_\alpha \frac{\partial}{\partial x^\nu} \left( -i\bar{\theta} \bar{\chi}(x) + \theta \sigma^\mu \bar{\theta} V_\mu + i\theta \theta \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i \bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) \right) \end{aligned} \right) \\ &\quad + \left( \begin{aligned} &2i(\theta \sigma^\nu)_\beta \frac{\partial}{\partial \bar{\theta}_\beta} \frac{\partial}{\partial x^\nu} \left( (\sigma^\mu \bar{\theta})_\alpha V_\mu + 2i\theta_\alpha \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i \bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) - i\bar{\theta} \theta \left( \lambda(x) - \frac{1}{2} i \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right)_\alpha \right. \\ &\quad \left. + \frac{1}{2} 2\theta_\alpha \bar{\theta} \theta \left( D(x) - \frac{1}{2} \frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu} \right) \right) \\ &2i \frac{\partial}{\partial \bar{\theta}^\beta} (\bar{\sigma}^\nu \theta)^\beta \frac{\partial}{\partial x^\nu} \left( -i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} (C(x) + i\theta \chi(x) - i\bar{\theta} \bar{\chi}(x) + \theta \sigma^\mu \bar{\theta} V_\mu) \right) \end{aligned} \right) \\ &\quad + \theta \theta \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} \left( i\chi_\alpha(x) + (\sigma^\mu \bar{\theta})_\alpha V_\mu - i\bar{\theta} \theta \left( \lambda(x) - \frac{1}{2} i \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right) - i(\sigma^\nu \bar{\theta})_\alpha \frac{\partial}{\partial x^\nu} (C(x) - i\bar{\theta} \bar{\chi}(x)) \right) \end{aligned}$$

1) 定数項

$$\begin{aligned}
& \left( -\frac{\partial}{\partial \bar{\theta}^\alpha} \frac{\partial}{\partial \theta_\alpha} \right) \left( -i \bar{\theta} \bar{\theta} \left( \lambda(x) - \frac{1}{2} i \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right)_\alpha - i (\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} (-i \bar{\theta} \bar{\chi}(x)) \right) \\
& \quad \leftarrow -i (\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} (-i \bar{\theta} \bar{\chi}(x)) = -(\sigma^\mu)_{\alpha\beta} \bar{\theta}^\beta \bar{\theta}_{\beta'} \frac{\partial \bar{\chi}^{\beta'}(x)}{\partial x^\mu} = -(\sigma^\mu)_{\alpha\beta} \left( -\frac{1}{2} \delta_{\beta'}^{\beta} \bar{\theta} \bar{\theta} \right) \frac{\partial \bar{\chi}^{\beta'}(x)}{\partial x^\mu} \\
& \quad = \frac{1}{2} \left( \sigma^\mu \frac{\partial \bar{\chi}^{\beta'}(x)}{\partial x^\mu} \right) \bar{\theta} \bar{\theta} \\
& = - \left( \frac{\partial}{\partial \bar{\theta}^\alpha} \frac{\partial}{\partial \theta_\alpha} \right) \left( \bar{\theta} \bar{\theta} \left( -i \lambda(x) - \frac{1}{2} \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right)_\alpha + \frac{1}{2} \left( \sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right) \bar{\theta} \bar{\theta} \right) \\
& = i \lambda_\alpha(x) \left( \frac{\partial}{\partial \bar{\theta}^\alpha} \frac{\partial}{\partial \theta_\alpha} \right) \bar{\theta} \bar{\theta} = 4 i \lambda_\alpha(x) \leftarrow \frac{\partial}{\partial \bar{\theta}^\alpha} \frac{\partial}{\partial \theta_\alpha} \bar{\theta} \bar{\theta} = 4
\end{aligned}$$

2)  $\theta$ -項

$$\begin{aligned}
& \left( -\frac{\partial}{\partial \bar{\theta}^\beta} \frac{\partial}{\partial \theta_\beta} \right) \left( \frac{1}{2} 2 \theta_\alpha \bar{\theta} \bar{\theta} \left( D - \frac{1}{2} \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} \right) - i (\sigma^\nu \bar{\theta})_\alpha \theta \sigma_\mu \bar{\theta} \frac{\partial V^\mu}{\partial x^\nu} \right) \\
& + \left( 2i \frac{\partial}{\partial \bar{\theta}^\alpha} (\bar{\sigma}^\nu \theta)^\alpha \frac{\partial}{\partial x^\nu} \right) \left( (\sigma^\mu \bar{\theta})_\alpha V_\mu - i (\sigma^\mu \bar{\theta})_\alpha \frac{\partial C}{\partial x^\mu} \right) \\
& \quad \leftarrow (\sigma^\nu \bar{\theta})_\alpha \theta \sigma_\mu \bar{\theta} = -(\sigma^\nu \bar{\theta})_\alpha \bar{\theta} \bar{\sigma}_\mu \theta = -(\sigma^\nu)_{\alpha\beta} \bar{\theta}^\beta \bar{\theta}_{\beta'} (\bar{\sigma}_\mu \theta)^{\beta'} = \frac{1}{2} (\sigma^\nu \bar{\sigma}_\mu \theta)_\alpha \bar{\theta} \bar{\theta} \\
& = \left( -\frac{\partial}{\partial \bar{\theta}^\beta} \frac{\partial}{\partial \theta_\beta} \right) \left( \theta_\alpha \bar{\theta} \bar{\theta} \left( D - \frac{1}{2} \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} \right) - i \frac{1}{2} (\sigma^\nu \bar{\sigma}_\mu \theta)_\alpha \bar{\theta} \bar{\theta} \frac{\partial V^\mu}{\partial x^\nu} \right) \\
& + \left( 2i \frac{\partial}{\partial \bar{\theta}^\beta} (\bar{\sigma}^\nu \theta)^\beta (\sigma^\mu \bar{\theta})_\alpha \right) \left( \frac{\partial V_\mu}{\partial x^\nu} - i \frac{\partial^2 C}{\partial x^\nu \partial x^\mu} \right) \leftarrow \frac{\partial}{\partial \bar{\theta}^\beta} \frac{\partial}{\partial \theta_\beta} \bar{\theta} \bar{\theta} = \frac{\partial}{\partial \bar{\theta}^\beta} \frac{\partial}{\partial \theta_\beta} \bar{\theta}_\alpha \bar{\theta}^\alpha = 4 \\
& \quad \leftarrow \frac{\partial}{\partial \bar{\theta}^\beta} (\bar{\sigma}^\nu \theta)^\beta (\sigma^\mu \bar{\theta})_\alpha = -\frac{\partial}{\partial \bar{\theta}^\beta} (\sigma^\mu \bar{\theta})_\alpha (\bar{\sigma}^\nu \theta)^\beta = -(\sigma^\mu)_{\alpha\beta} (\bar{\sigma}^\nu \theta)^\beta = -(\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \\
& = (-4) \left( \theta_\alpha \left( D - \frac{1}{2} \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} \right) - i \frac{1}{2} (\sigma^\nu \bar{\sigma}_\mu \theta)_\alpha \frac{\partial V^\mu}{\partial x^\nu} \right) + 2i (-\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \left( \frac{\partial V_\mu}{\partial x^\nu} - i \frac{\partial^2 C}{\partial x^\nu \partial x^\mu} \right) \\
& = -4 \theta_\alpha \left( D - \frac{1}{2} \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} \right) + 2i (\sigma^\nu \bar{\sigma}^\mu \theta)_\alpha \frac{\partial V_\mu}{\partial x^\nu} - 2i (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \left( \frac{\partial V_\mu}{\partial x^\nu} - i \frac{\partial^2 C}{\partial x^\nu \partial x^\mu} \right) \\
& = -4 \theta_\alpha D + 2 \theta_\alpha \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} + 2i (\sigma^\nu \bar{\sigma}^\mu \theta)_\alpha \frac{\partial V_\mu}{\partial x^\nu} - 2i (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \frac{\partial V_\mu}{\partial x^\nu} - 2 (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \frac{\partial^2 C}{\partial x^\nu \partial x^\mu} \\
& = -4 \theta_\alpha D + 2 \theta_\alpha \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} + 2i \left[ (\sigma^\nu \bar{\sigma}^\mu \theta)_\alpha - (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \right] \frac{1}{2} \left( \frac{\partial V_\mu}{\partial x^\nu} - \frac{\partial V_\nu}{\partial x^\mu} \right) - 2 (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha \frac{\partial^2 C}{\partial x^\nu \partial x^\mu} \\
& \quad \leftarrow \sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu = 2i \sigma^{\mu\nu} \quad \& \quad \sigma^\mu \bar{\sigma}^\nu = g^{\mu\nu} + 2i \sigma^{\mu\nu} \\
& = -4 \theta_\alpha D + 2 \theta_\alpha \frac{\partial^2 C}{\partial x^\mu \partial x_\mu} + 2i (-2i \sigma^{\mu\nu} \theta)_\alpha \frac{1}{2} (-F_{\mu\nu}) - 2 (g^{\mu\nu} \theta)_\alpha \frac{\partial^2 C}{\partial x^\nu \partial x^\mu} \\
& = -4 \theta_\alpha D - 2 (\sigma^{\mu\nu} \theta)_\alpha F_{\mu\nu} = - \left[ 4D \delta_\alpha^\beta + 2 (\sigma^{\mu\nu})_{\alpha\bar{\alpha}}^\beta F_{\mu\nu} \right] \theta_\beta
\end{aligned}$$

3)  $\theta\theta$ -項

$$\begin{aligned}
& \left( -\frac{\partial}{\partial\bar{\theta}^\beta} \frac{\partial}{\partial\theta_\beta} \right) \left( -i(\sigma^v\bar{\theta})_\alpha \frac{\partial}{\partial x^v} \bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial\chi(x)}{\partial x^\mu} \right) \right) i\theta\theta \\
& + \left( 2i \frac{\partial}{\partial\bar{\theta}^\beta} (\bar{\sigma}^v\theta)^\beta \frac{\partial}{\partial x^v} \right) \left( 2i\theta_\alpha \left( \bar{\theta}\bar{\lambda}(x) - \frac{1}{2} i\bar{\theta}\bar{\sigma}^\mu \frac{\partial\chi(x)}{\partial x^\mu} \right) - i(\sigma^\mu\bar{\theta})_\alpha \frac{\partial}{\partial x^\mu} i\theta\chi(x) \right) + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta \\
& = \left( -\frac{\partial}{\partial\bar{\theta}^\beta} \frac{\partial}{\partial\theta_\beta} \right) \left( (\sigma^v\bar{\theta})_\alpha \bar{\theta} \left( \frac{\partial\bar{\lambda}}{\partial x^v} - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right) \right) \theta\theta \\
& - \left( 2i \frac{\partial}{\partial\bar{\theta}^\beta} \right) \left( 2i \left( \bar{\theta} \frac{\partial\bar{\lambda}}{\partial x^v} + \frac{1}{2} i \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \sigma^\mu\bar{\theta} \right) \theta_\alpha + (\sigma^\mu\bar{\theta})_\alpha \frac{\partial^2}{\partial x^v\partial x^\mu} \theta\chi(x) \right) (\bar{\sigma}^v\theta)^\beta + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta \\
& \Leftarrow (\sigma^v\bar{\theta})_\alpha \bar{\theta}\bar{\varepsilon} = (\sigma^v)_{\alpha\alpha'} \bar{\theta}^{\alpha'} \bar{\theta}_\beta \bar{\varepsilon}^{\beta'} = -\frac{1}{2} (\sigma^v\bar{\varepsilon})_\alpha \bar{\theta}\bar{\theta} \\
& = \left( -\frac{\partial}{\partial\bar{\theta}^\beta} \frac{\partial}{\partial\theta_\beta} \right) \left( -\frac{1}{2} \left( \sigma^v \left( \frac{\partial\bar{\lambda}}{\partial x^v} - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right) \right)_\alpha \bar{\theta}\bar{\theta} \right) \theta\theta \\
& - (2i) \left( 2i \left( -\frac{\partial\bar{\lambda}}{\partial x^v} - \frac{1}{2} i \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \sigma^\mu \right)_\beta \theta_\alpha + (\sigma^\mu)_{\alpha\beta} \frac{\partial^2}{\partial x^v\partial x^\mu} \theta\chi(x) \right) (\bar{\sigma}^v\theta)^\beta + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta \\
& = (-4) \left( -\frac{1}{2} \right) \left( \sigma^v \left( \frac{\partial\bar{\lambda}}{\partial x^v} - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right) \right)_\alpha \theta\theta \\
& - (2i) \left( i \left( \frac{\partial\bar{\lambda}}{\partial x^v} \bar{\sigma}^v\theta + \frac{1}{2} i \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \sigma^\mu \bar{\sigma}^v\theta \right) 2\theta_\alpha + (\sigma^\mu \bar{\sigma}^v\theta)_\alpha \frac{\partial^2}{\partial x^v\partial x^\mu} \theta\chi(x) \right) + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta \\
& \Leftarrow \bar{\lambda}\bar{\sigma}^v\theta\theta_\alpha = -\theta\sigma^v\bar{\lambda}\theta_\alpha = -\theta^\beta (\sigma^v\bar{\lambda})_\beta \theta_\alpha = (\sigma^v\bar{\lambda})_\beta \theta^\beta\theta_\alpha = \frac{1}{2} (\sigma^v\bar{\lambda})_\alpha \theta\theta \\
& \Leftarrow \chi\sigma^\mu\bar{\sigma}^v\theta\theta_\alpha = \theta\sigma^v\bar{\sigma}^\mu\chi\theta_\alpha = \theta^\beta (\sigma^v\bar{\sigma}^\mu\chi)_\beta \theta_\alpha = -(\sigma^v\bar{\sigma}^\mu\chi)_\beta \theta^\beta\theta_\alpha = -\frac{1}{2} (\sigma^v\bar{\sigma}^\mu\chi)_\alpha \theta\theta \\
& \Leftarrow (\sigma^\mu\bar{\sigma}^v\theta)_\alpha \theta\chi = (\sigma^\mu\bar{\sigma}^v)_{\alpha\beta} \theta_\beta \theta^{\beta'} \chi_{\beta'} = -(\sigma^\mu\bar{\sigma}^v)_{\alpha\beta} \chi_{\beta'} \theta^{\beta'}\theta_\beta = -\frac{1}{2} (\sigma^\mu\bar{\sigma}^v\chi)_\alpha \theta\theta \\
& = 2 \left( \sigma^v \left( \frac{\partial\bar{\lambda}}{\partial x^v} - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right) \right)_\alpha \theta\theta + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta \\
& - (2i) \left( 2i \left( \frac{1}{2} \left( \sigma^v \frac{\partial\bar{\lambda}}{\partial x^v} \right)_\alpha \theta\theta + \frac{1}{2} i \left( -\frac{1}{2} \sigma^v\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right)_\alpha \theta\theta \right) - \frac{1}{2} \left( \sigma^\mu\bar{\sigma}^v \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right)_\alpha \theta\theta \right) \\
& = \left( \sigma^v 2 \frac{\partial\bar{\lambda}}{\partial x^v} - i\sigma^v\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right)_\alpha \theta\theta + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta \\
& + \left( 2 \left( \sigma^v \frac{\partial\bar{\lambda}}{\partial x^v} \right)_\alpha - i \left( \sigma^v\bar{\sigma}^\mu \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right)_\alpha + i \left( \sigma^\mu\bar{\sigma}^v \frac{\partial^2\chi}{\partial x^v\partial x^\mu} \right)_\alpha \right) \theta\theta \\
& \Leftarrow \sigma^v\bar{\sigma}^\mu = g^{v\mu} + i\sigma^{v\mu} \\
& = \left( 2\sigma^v \frac{\partial\bar{\lambda}}{\partial x^v} - i \frac{\partial^2\chi}{\partial x^\mu\partial x_\mu} \right)_\alpha \theta\theta + i \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \theta\theta + \left( 2 \left( \sigma^v \frac{\partial\bar{\lambda}}{\partial x^v} \right)_\alpha - i \left( \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \right)_\alpha + i \left( \frac{\partial^2\chi_\alpha}{\partial x^\mu\partial x_\mu} \right)_\alpha \right) \theta\theta = 4 \left( \sigma^v \frac{\partial\bar{\lambda}}{\partial x^v} \right)_\alpha \theta\theta
\end{aligned}$$

以上から、

$$W_\alpha(x) \equiv \bar{D}_\beta \bar{D}^\beta D_\alpha V(x) = 4i\lambda_\alpha(x) - \left[ 4D(x)\delta_\alpha^\beta + 2(\sigma^{\mu\nu})_{\bar{\alpha}}^\beta F_{\mu\nu}(x) \right] \theta_\beta + 4 \left( \sigma^\nu \frac{\partial \bar{\lambda}(x)}{\partial x^\nu} \right)_\alpha \theta\theta + (\dots)$$

つまり、

$$\begin{aligned} W_\alpha(y) &\equiv \bar{D}_\beta \bar{D}^\beta D_\alpha V(x) = 4i\lambda_\alpha(y) - \left[ 4D(y)\delta_\alpha^\beta + 2(\sigma^{\mu\nu})_{\bar{\alpha}}^\beta F_{\mu\nu}(y) \right] \theta_\beta + 4 \left( \sigma^\nu \frac{\partial \bar{\lambda}(y)}{\partial x^\nu} \right)_\alpha \theta\theta \\ V(x, \theta, \bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta m(x) - i\bar{\theta}\bar{\theta} m^\dagger(x) + \theta\sigma^\mu \bar{\theta} V_\mu \\ &\quad + i\theta\theta\bar{\theta} \left( \bar{\lambda}(x) - \frac{1}{2} i\bar{\sigma}^\mu \frac{\partial \chi(x)}{\partial x^\mu} \right) - i\bar{\theta}\bar{\theta}\theta \left( \lambda(x) - \frac{1}{2} i\sigma^\mu \frac{\partial \bar{\chi}(x)}{\partial x^\mu} \right) + \frac{1}{2} \theta\theta\bar{\theta}\bar{\theta} \left( D(x) - \frac{1}{2} \frac{\partial^2 C(x)}{\partial x^\mu \partial x_\mu} \right) \\ \text{with } D_\alpha &= \frac{\partial}{\partial \theta^\alpha} - i(\sigma^\mu \bar{\theta})_\alpha \frac{\partial}{\partial x^\mu}, \quad \bar{D}_\alpha = -\frac{\partial}{\partial \bar{\theta}^\alpha} + i(\theta\sigma^\mu)_\alpha \frac{\partial}{\partial x^\mu} \\ y^\mu &= x^\mu - i\theta\sigma^\mu \bar{\theta} \end{aligned}$$

が求めるカイラルゲージ超場になる。