

第0章 表記法

metric

$$g_{\mu\nu} = \text{diag.}(1, -1, -1, -1)$$

totally antisymmetrized tensor

$$\varepsilon_{0123} = 1$$

$SL(2, \mathbf{C})$ algebra

$$\begin{aligned} \{\gamma_\mu, \gamma_\nu\} &= 2g_{\mu\nu} & \gamma_\mu &= \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ \sigma_\mu &= (1, -\boldsymbol{\sigma})_{\alpha\beta} & \bar{\sigma}_\mu &= (1, \boldsymbol{\sigma})^{\alpha\beta} & \boldsymbol{\sigma} &= (\sigma^1, \sigma^2, \sigma^3) \\ \sigma^\mu &= (1, \boldsymbol{\sigma})_{\alpha\beta} & \bar{\sigma}^\mu &= (1, -\boldsymbol{\sigma})^{\alpha\beta} & \{\sigma^i, \sigma^j\} &= 2\delta^{ij} \end{aligned}$$

two component spinor

$$\begin{aligned} \Psi_\alpha &= \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^\alpha \end{pmatrix} & \psi^\alpha &= \varepsilon^{\alpha\beta} \psi_\beta = (-i\sigma^2 \psi)^\alpha & \bar{\psi}^\alpha (= \psi^{\alpha*}) &= \varepsilon^{\alpha\beta} \bar{\psi}_\beta = (i\psi^\dagger \sigma^2)^\alpha \\ & & \psi_\alpha &= \varepsilon_{\alpha\beta} \psi^\beta = (i\sigma^2 \psi)_\alpha & \bar{\psi}_\alpha (= \psi_\alpha^*) &= \varepsilon_{\alpha\beta} \bar{\psi}^\beta = (-i\psi^\dagger \sigma^2)_\alpha \\ \varepsilon_{12} &= -\varepsilon_{21} = -1 & \varepsilon^{12} &= -\varepsilon^{21} = 1 \end{aligned}$$

spinor convention

$$\begin{aligned} \psi\chi &(\equiv \varepsilon^{\alpha\beta} \psi_\beta \chi_\alpha) = \psi^\alpha \chi_\alpha = -\chi_\alpha \psi^\alpha = \chi^\alpha \psi_\alpha = \chi\psi \\ \bar{\psi}\bar{\chi} &(\equiv -\varepsilon^{\alpha\beta} \bar{\psi}_\beta \bar{\chi}_\alpha) = \bar{\psi}_\alpha \bar{\chi}^\alpha = -\bar{\chi}^\alpha \bar{\psi}_\alpha = \bar{\chi}_\alpha \bar{\psi}^\alpha = \bar{\chi}\bar{\psi} \\ (*) \bar{\psi}\bar{\chi} &= (\psi\chi)^\dagger = \varepsilon^{\alpha\beta} \bar{\chi}_\alpha \bar{\psi}_\beta = -\varepsilon^{\alpha\beta} \bar{\psi}_\beta \bar{\chi}_\alpha \end{aligned}$$

useful relations

$$\begin{aligned} 1) \quad \sigma_\mu \bar{\sigma}_\nu &= g_{\mu\nu} + i\sigma_{\mu\nu} & [\gamma_\mu, \gamma_\nu] &= \begin{pmatrix} 2i\sigma_{\mu\nu} & 0 \\ 0 & 2i\bar{\sigma}_{\mu\nu} \end{pmatrix} \\ \bar{\sigma}_\mu \sigma_\nu &= g_{\mu\nu} + i\bar{\sigma}_{\mu\nu} & & \\ 2i\sigma_{\mu\nu} &= \sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu & 2g_{\mu\nu} &= \sigma_\mu \bar{\sigma}_\nu + \sigma_\nu \bar{\sigma}_\mu \quad \text{添え字のつき方 : } (\sigma_{\mu\nu})_\alpha{}^\beta \\ 2i\bar{\sigma}_{\mu\nu} &= \bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu & 2g_{\mu\nu} &= \bar{\sigma}_\mu \sigma_\nu + \bar{\sigma}_\nu \sigma_\mu \quad \text{添え字のつき方 : } (\bar{\sigma}_{\mu\nu})^\alpha{}_\beta \\ 2) \quad \sigma_\mu \bar{\sigma}_\rho \sigma_\nu &+ \sigma_\nu \bar{\sigma}_\rho \sigma_\mu = 2(g_{\mu\rho} \sigma_\nu + g_{\rho\nu} \sigma_\mu - g_{\mu\nu} \sigma_\rho) \\ \bar{\sigma}_\mu \sigma_\rho \bar{\sigma}_\nu &+ \bar{\sigma}_\nu \sigma_\rho \bar{\sigma}_\mu = 2(g_{\mu\rho} \bar{\sigma}_\nu + g_{\rho\nu} \bar{\sigma}_\mu - g_{\mu\nu} \bar{\sigma}_\rho) \\ \sigma_\mu \bar{\sigma}_\rho \sigma_\nu - \sigma_\nu \bar{\sigma}_\rho \sigma_\mu &= -2i\varepsilon_{\mu\rho\nu\tau} \sigma^\tau \\ \bar{\sigma}_\mu \sigma_\rho \bar{\sigma}_\nu - \bar{\sigma}_\nu \sigma_\rho \bar{\sigma}_\mu &= 2i\varepsilon_{\mu\rho\nu\tau} \bar{\sigma}^\tau \\ 3) \quad \theta^\alpha \theta_\beta &= \frac{1}{2} \delta_\beta^\alpha (\theta\theta) & \bar{\theta}_\alpha \bar{\theta}^\beta &= \frac{1}{2} \delta_\beta^\alpha (\bar{\theta}\bar{\theta}) \\ \theta^\alpha \bar{\theta}^\beta &= \frac{1}{2} (\bar{\sigma}^\mu)^{\beta\alpha} (\theta\sigma^\mu\bar{\theta}) & \bar{\theta}_\alpha \theta_\beta &= \frac{1}{2} (\sigma_\mu)_{\beta\alpha} (\bar{\theta}\bar{\sigma}^\mu\theta) \\ \theta\alpha \cdot \theta\beta &= -\frac{1}{2} (\theta\theta)(\alpha\beta) \end{aligned}$$

$$4) \theta \sigma_\mu \bar{\alpha} = -\bar{\alpha} \bar{\sigma}_\mu \theta \quad \theta \alpha \bar{\theta} \bar{\alpha} = \frac{1}{2} (\alpha \sigma_\mu \bar{\beta}) (\theta \sigma^\mu \bar{\theta})$$

$$\theta \sigma_\mu \bar{\alpha} \cdot \theta \sigma_\nu \bar{\alpha} = \frac{1}{2} (\theta \theta) (\bar{\alpha} \bar{\alpha}) g_{\mu\nu}$$

$$5) \theta \sigma^\mu \bar{\sigma}^\nu \theta = \theta \sigma^\nu \bar{\sigma}^\mu \theta = g^{\mu\nu} (\theta \theta) \quad \bar{\theta} \bar{\sigma}^\mu \sigma^\nu \bar{\theta} = \bar{\theta} \bar{\sigma}^\nu \sigma^\mu \bar{\theta} = g^{\mu\nu} (\bar{\theta} \bar{\theta})$$

$$\theta \sigma^\mu \bar{\sigma}^\nu \alpha = \alpha \sigma^\nu \bar{\sigma}^\mu \theta \bar{\theta} \bar{\sigma}^\mu \sigma^\nu \bar{\alpha} = \bar{\alpha} \bar{\sigma}^\nu \sigma^\mu \bar{\theta}$$

$$\theta \sigma^{\mu\nu} \alpha = -\alpha \sigma^{\mu\nu} \theta$$

$$\theta \sigma_\mu \bar{\sigma}_{\nu\rho} \bar{\alpha} = \bar{\alpha} \bar{\sigma}_{\nu\rho} \bar{\sigma}_\mu \theta$$

$$\theta \sigma_{\nu\rho} \sigma_\mu \bar{\alpha} = \bar{\alpha} \bar{\sigma}_\mu \sigma_{\nu\rho} \theta$$

$$\theta \sigma_\mu \bar{\sigma}_\nu \sigma_\rho \bar{\alpha} = -\bar{\alpha} \bar{\sigma}_\rho \sigma_\nu \bar{\sigma}_\mu \theta$$

$$6) (\theta \sigma_\mu \bar{\alpha})^\dagger = \alpha \sigma_\mu \bar{\theta} = -\bar{\theta} \bar{\sigma}_\mu \alpha \quad (\sigma^{\mu\nu})^\dagger = \bar{\sigma}^{\mu\nu}$$

$$(\theta \sigma^\mu \bar{\sigma}^\nu \alpha)^\dagger = \bar{\alpha} \bar{\sigma}^\nu \sigma^\mu \bar{\theta} \quad (\theta \sigma^{\mu\nu} \alpha)^\dagger = -\bar{\alpha} \bar{\sigma}^{\mu\nu} \bar{\theta}$$

$$7) (\sigma^\mu)_{\alpha\beta} = (\sigma^2 \bar{\sigma}^{\mu T} \sigma^2)_{\alpha\beta} = (\sigma^2)_{\alpha\alpha'} (\bar{\sigma}^{\mu T})^{\alpha'\beta'} (\sigma^2)_{\beta'\beta}$$

$$8) \text{Tr}(\sigma_{\mu\nu} \sigma_{\rho\sigma}) = 2(g_{\mu\rho} g_{\nu\sigma} - g_{\nu\rho} g_{\mu\sigma} + i\varepsilon_{\mu\nu\rho\sigma})$$

$$\text{Tr}(\sigma^\mu \bar{\sigma}^\nu \sigma^\rho \bar{\sigma}^\sigma) = 2(g^{\mu\sigma} g^{\nu\rho} + g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + i\varepsilon^{\mu\nu\rho\sigma})$$

$$9) \sigma_{\mu\nu} \sigma_\rho - \sigma_\rho \bar{\sigma}_{\mu\nu} = 2i(g_{\mu\rho} \sigma_\nu - g_{\nu\rho} \sigma_\mu)$$

$$\bar{\sigma}_{\mu\nu} \bar{\sigma}_\rho - \bar{\sigma}_\rho \sigma_{\mu\nu} = 2i(g_{\mu\rho} \bar{\sigma}_\nu - g_{\nu\rho} \bar{\sigma}_\mu)$$

$$10) \sigma^{\mu\nu} \sigma^\rho = \frac{1}{i} (g^{\nu\rho} \sigma^\mu - g^{\mu\rho} \sigma^\nu - i\varepsilon^{\mu\nu\rho\sigma} \sigma_\sigma)$$

$$\sigma^\rho \bar{\sigma}^{\mu\nu} = \frac{1}{i} (g^{\rho\mu} \sigma^\nu - g^{\rho\nu} \sigma^\mu - i\varepsilon^{\rho\mu\nu\sigma} \sigma_\sigma)$$

$$\bar{\sigma}^{\mu\nu} \bar{\sigma}^\rho = \frac{1}{i} (g^{\nu\rho} \bar{\sigma}^\mu - g^{\mu\rho} \bar{\sigma}^\nu - i\varepsilon^{\mu\nu\rho\sigma} \bar{\sigma}_\sigma)$$

$$\bar{\sigma}^\rho \sigma^{\mu\nu} = \frac{1}{i} (g^{\rho\mu} \bar{\sigma}^\nu - g^{\rho\nu} \bar{\sigma}^\mu - i\varepsilon^{\rho\mu\nu\sigma} \bar{\sigma}_\sigma)$$

$$11) \varepsilon^{\mu\nu\rho\sigma} \psi \sigma_\mu \bar{\sigma}_\nu \sigma_\rho \bar{\psi} = 3! i \psi \sigma^\sigma \bar{\psi}$$

$$\varepsilon^{\mu\nu\rho\sigma} \bar{\psi} \bar{\sigma}_\mu \sigma_\nu \bar{\sigma}_\rho \psi = -3! i \bar{\psi} \bar{\sigma}^\sigma \psi$$

グラスマン数の積分

$$\int d\theta^\alpha = \int d\bar{\theta}^\alpha = 0, \quad \int d\theta^\alpha \theta^\alpha = \int d\bar{\theta}^\alpha \bar{\theta}^\alpha = 1 \quad (\text{with no summation over } \alpha)$$

$$d^2\theta = -\frac{1}{4} d\theta^\alpha d\theta_\alpha \text{ so that } \int d^2\theta \theta\theta = 1, \quad d^2\bar{\theta} = -\frac{1}{4} d\bar{\theta}_\alpha d\bar{\theta}^\alpha \text{ so that } \int d^2\bar{\theta} \bar{\theta}\bar{\theta} = 1$$

$$d^4\theta = d^2\theta d^2\bar{\theta}$$