第8回：角運動量と回転軸

必要になる基礎知識

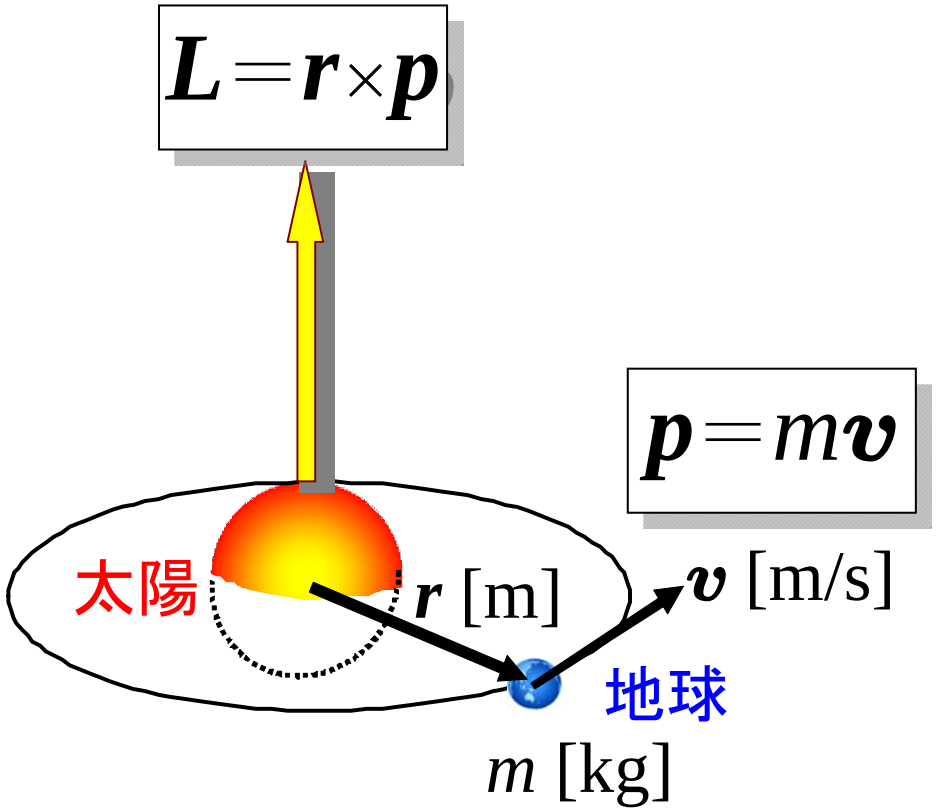
⊙$\boldsymbol{L}(t)=\boldsymbol{r}(t)\times\boldsymbol{p}(t)$のとき$\dfrac{d\boldsymbol{L}(t)}{dt}$は？

⊙$\dfrac{d\boldsymbol{p}(t)}{dt}$が力になるか？

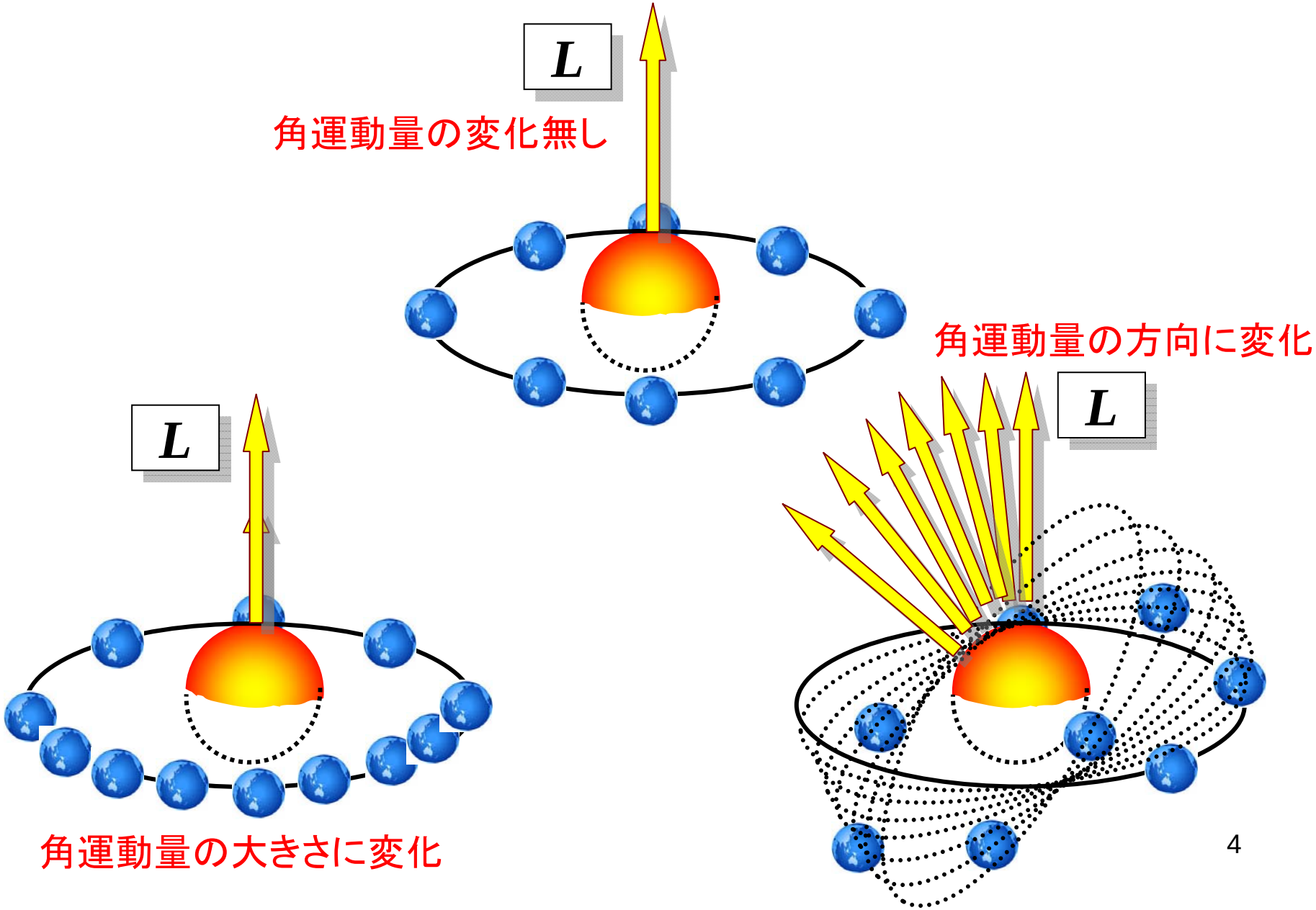
⊙$\boldsymbol{A}\times\boldsymbol{A}$は？

角運動量

$$\boxed{\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}}$$



角運動量の時間変化



角運動量の時間変化の数値化

角運動量ベクトルの時間変化分 $= \boldsymbol{L}(t+\delta t)-\boldsymbol{L}(t)$

$$\boldsymbol{L}(t)=\boldsymbol{r}(t)\times\boldsymbol{p}(t)$$

$$\boldsymbol{L}\left(t+\overbrace{\delta t}^{\text{微小変化分}}\right)\fallingdotseq\boldsymbol{L}(t)+\overbrace{\delta t}^{\text{微小変化分}}\frac{d\boldsymbol{L}(t)}{dt}\Rightarrow\boldsymbol{L}(t+\delta t)-\boldsymbol{L}(t)=\delta t\frac{d\boldsymbol{L}(t)}{dt}$$

角運動量ベクトルの微小時間変化分 $= \overbrace{\delta t}^{\text{微小変化分}}\dfrac{d\boldsymbol{L}(t)}{dt}$

時間と共に変化する $\Rightarrow \boldsymbol{L}(t+\delta t)\neq\boldsymbol{L}(t)\Rightarrow\dfrac{d\boldsymbol{L}(t)}{dt}\neq\overbrace{\boldsymbol{0}}^{\text{零ベクトル}}$

$$\frac{d\boldsymbol{L}(t)}{dt}=\frac{d}{dt}\left[\overbrace{\boldsymbol{r}(t)}^{\boldsymbol{a}}\times\overbrace{\boldsymbol{p}(t)}^{\boldsymbol{b}}\right]=\frac{d\overbrace{\boldsymbol{r}(t)}^{\boldsymbol{a}}}{dt}\times\overbrace{\boldsymbol{p}(t)}^{\boldsymbol{b}}+\overbrace{\boldsymbol{r}(t)}^{\boldsymbol{a}}\times\frac{d\overbrace{\boldsymbol{p}(t)}^{\boldsymbol{b}}}{dt}$$

【公式】$\dfrac{d}{dt}(\boldsymbol{a}\times\boldsymbol{b})=\boldsymbol{a}\times\dfrac{d\boldsymbol{b}}{dt}+\dfrac{d\boldsymbol{a}}{dt}\times\boldsymbol{b}$

$$\frac{d\boldsymbol{L}(t)}{dt}=\frac{d\boldsymbol{r}(t)}{dt}\times\boldsymbol{p}(t)+\boldsymbol{r}(t)\times\frac{d\boldsymbol{p}(t)}{dt}$$

$$\boxed{\frac{d\boldsymbol{L}(t)}{dt}=\frac{d\boldsymbol{r}(t)}{dt}\times\boldsymbol{p}(t)+\boldsymbol{r}(t)\times\frac{d\boldsymbol{p}(t)}{dt}}$$

$$\frac{d\boldsymbol{L}(t)}{dt}=\frac{d\boldsymbol{r}(t)}{dt}\times\boxed{\boldsymbol{p}(t)}+\boldsymbol{r}(t)\times\frac{d\boxed{\boldsymbol{p}(t)}}{dt}$$

$$\boxed{\boldsymbol{p}(t)=m\boldsymbol{v}(t)}$$

$$\frac{d\boldsymbol{r}(t)}{dt}=\boldsymbol{v}(t)$$

$$\frac{d\boldsymbol{p}(t)}{dt}=\frac{d\left[m\boldsymbol{v}(t)\right]}{dt}=\overbrace{m}^{\text{質量}}\overbrace{\frac{d\boldsymbol{v}(t)}{dt}}^{\text{加速度}}=\overbrace{\boldsymbol{F}(t)}^{\text{力}}$$

$$=\overbrace{\boldsymbol{v}(t)}^{\boldsymbol{A}\text{ベクトル}}\times m\overbrace{\boldsymbol{v}(t)}^{\boldsymbol{A}\text{ベクトル}}+\boldsymbol{r}(t)\times\boldsymbol{F}(t)$$

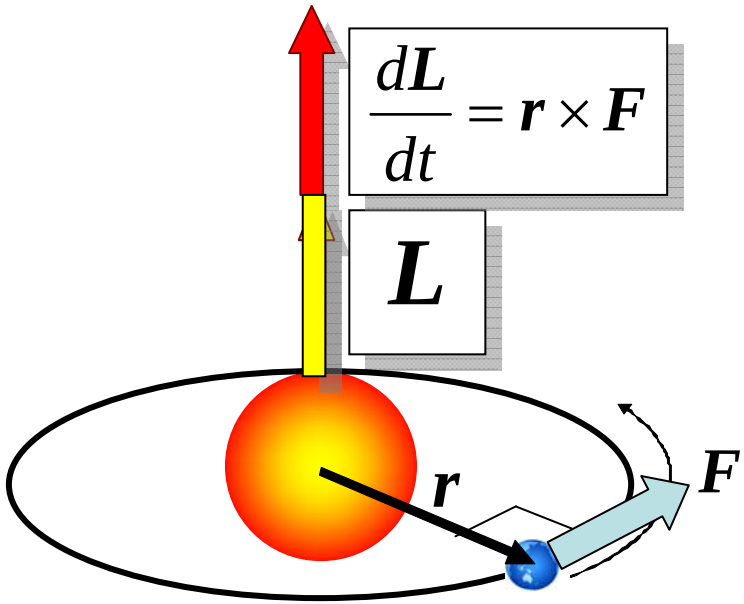
【公式】$\boldsymbol{A}\times\boldsymbol{A}=\overbrace{\boldsymbol{0}}^{\text{零ベクトル}}$

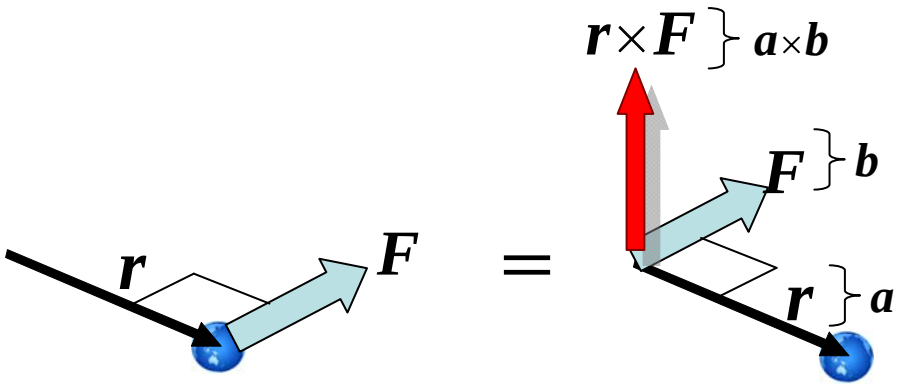
$$=\boldsymbol{r}(t)\times\boldsymbol{F}(t)$$

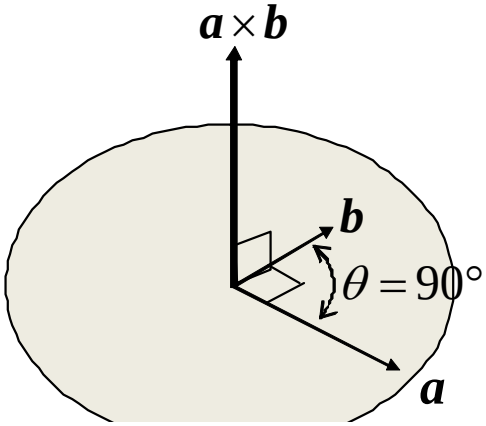
$$\boxed{\frac{d\boldsymbol{L}(t)}{dt}=\boldsymbol{r}(t)\times\boldsymbol{F}(t)\left(=\boldsymbol{\tau}(t)\text{: トルクベクトル}\right)}$$

角運動量の変化と力の方向

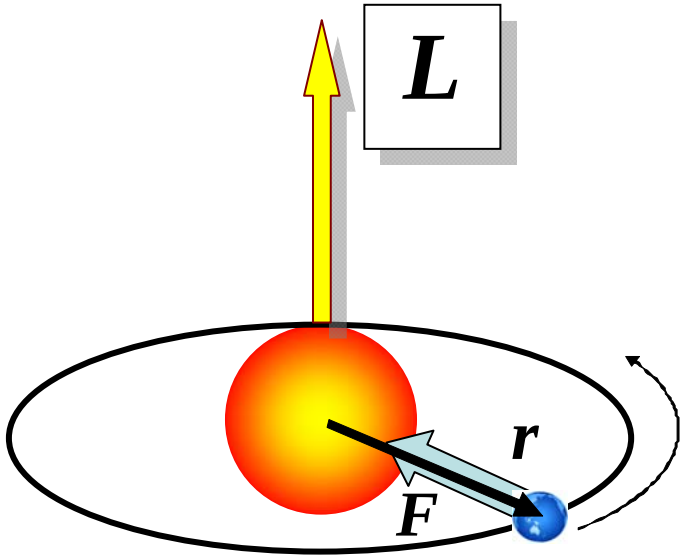
$$\frac{d\boldsymbol{L}(t)}{dt} = \boldsymbol{r}(t) \times \boldsymbol{F}(t) \neq \overset{\text{零ベクトル}}{\boldsymbol{0}} \quad \text{の時}$$



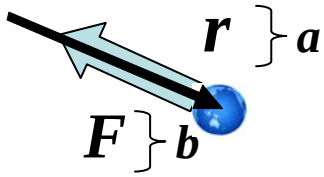




$$\frac{d\boldsymbol{L}(t)}{dt} = \boldsymbol{r}(t) \times \boldsymbol{F}(t) = \overbrace{\boldsymbol{0}}^{\text{零ベクトル}} \quad \text{の時}$$

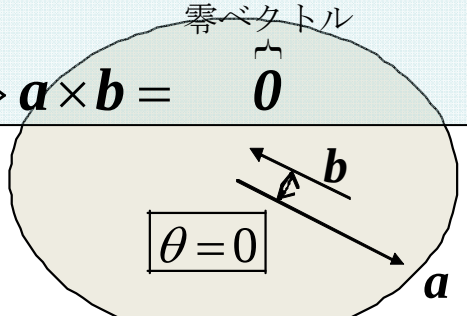


$$\overbrace{\boldsymbol{r}}^{\boldsymbol{a}} \times \overbrace{\boldsymbol{F}}^{\boldsymbol{b}} = \overbrace{\boldsymbol{0}}^{\text{零ベクトル}}$$



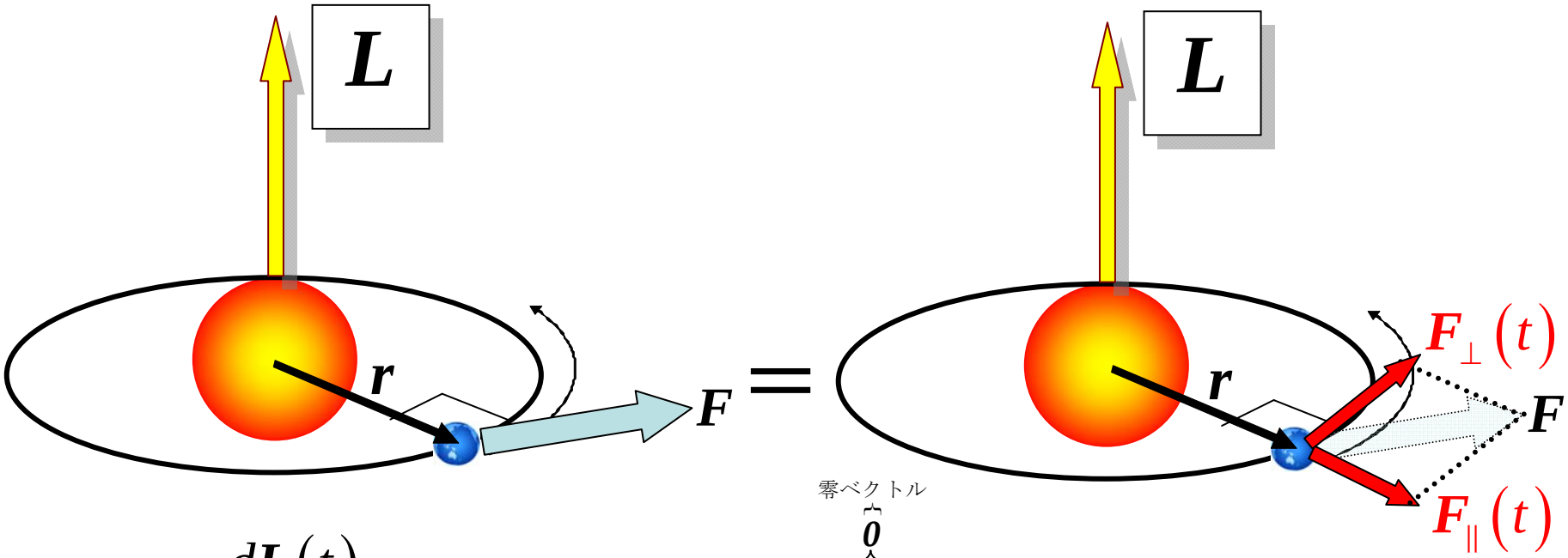
$$|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \sin\theta \overbrace{=}^{\sin\theta=0} 0$$

$$\Rightarrow \boldsymbol{a} \times \boldsymbol{b} = \overbrace{\boldsymbol{0}}^{\text{零ベクトル}}$$



角運動量の変化と力の方向

$$\frac{d\boldsymbol{L}(t)}{dt} = \boldsymbol{r}(t) \times \boldsymbol{F}(t)$$

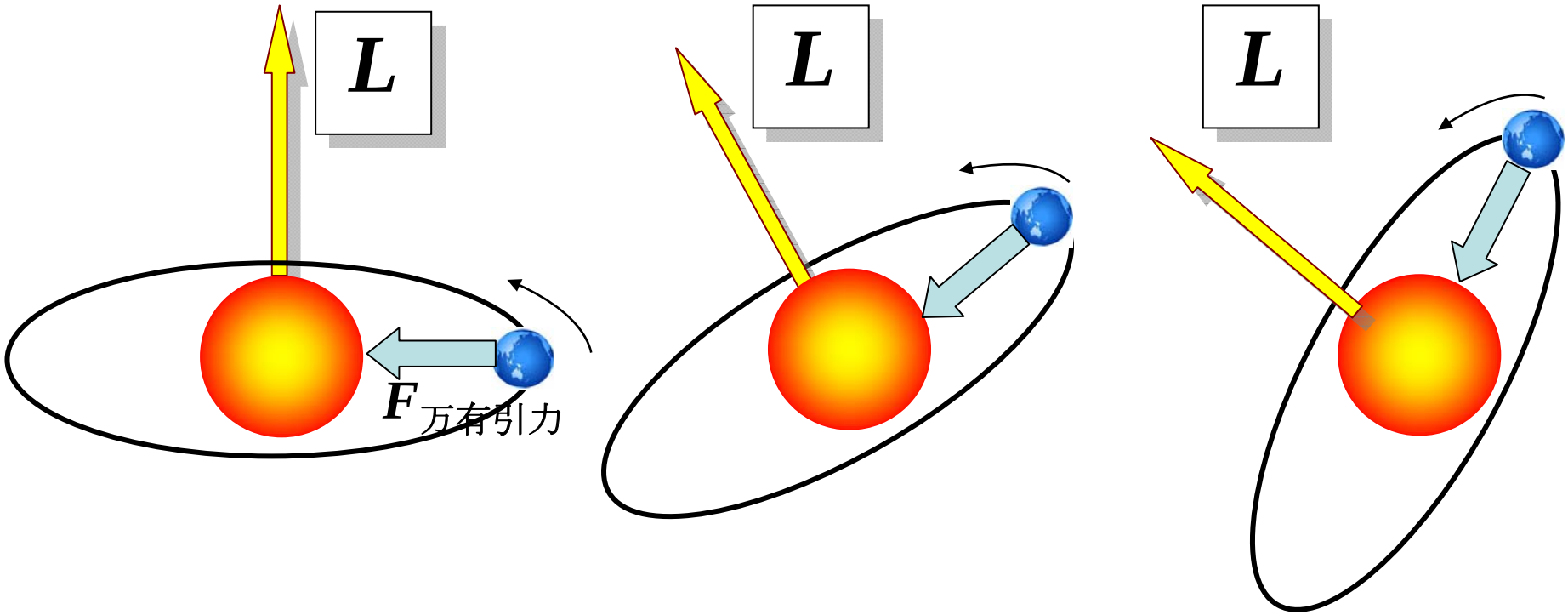


$$\frac{d\boldsymbol{L}(t)}{dt} = \boldsymbol{r}(t) \times \boldsymbol{F}(t) = \overbrace{\boldsymbol{r}(t) \times \boldsymbol{F}_{\parallel}(t)}^{\text{零ベクトル}\ \boldsymbol{0}} + \boldsymbol{r}(t) \times \boldsymbol{F}_{\perp}(t)$$

$$\frac{d\boldsymbol{L}(t)}{dt} = \boldsymbol{r}(t) \times \boldsymbol{F}_{\perp}(t)$$

地球公転面の変化と万有引力

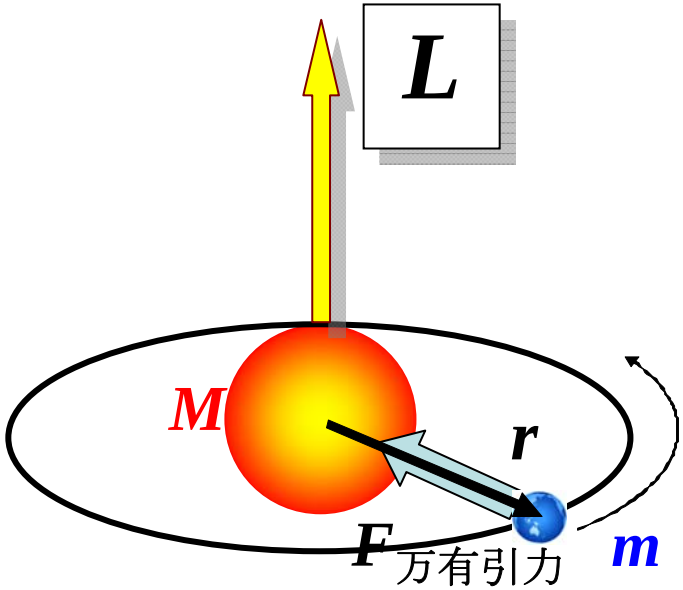
実は、太陽系での地球公転面の年変化というのがあって・・・



現在　　5億年前　　10億年前

角運動量の変化があるか調べればよい

【例】角運動量の変化と万有引力



現在

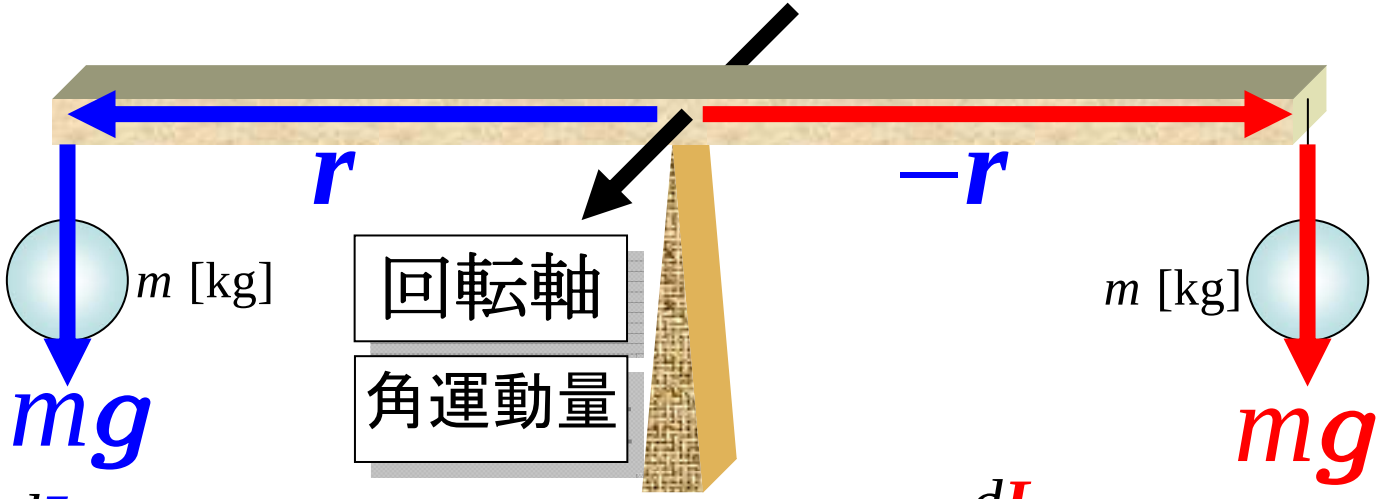
$$\boldsymbol{F}_{\text{万有引力}} = -\overbrace{G}^{\text{ニュートン定数}} \frac{Mm}{r^2} \overbrace{\frac{\boldsymbol{r}}{r}}^{\text{半径}(\boldsymbol{r})\text{方向の単位ベクトル}}$$

$$\frac{d\boldsymbol{L}(t)}{dt} = \boldsymbol{r}(t) \times \boldsymbol{F}_{\text{万有引力}}(t) = \overbrace{\boldsymbol{0}}^{\text{零ベクトル}}$$

5億年前も同じ

10億年前も同じ

【例】角運動量変化と釣り合い



$$\frac{d\boldsymbol{L}}{dt} = \boldsymbol{r} \times mg$$

$$\frac{d\boldsymbol{L}}{dt} = (-\boldsymbol{r}) \times mg$$

全角運動量：$\boldsymbol{L} = \boldsymbol{L} + \boldsymbol{L}$

$$\frac{d\boldsymbol{L}}{dt} = \frac{d\boldsymbol{L}}{dt} + \frac{d\boldsymbol{L}}{dt} = \boldsymbol{r} \times mg + (-\boldsymbol{r}) \times mg = \overbrace{\boldsymbol{0}}^{\text{零ベクトル}}$$

従って、角運動量は**大きさ**も回転**方向**も**変らない**

釣り合いをずっと保つ事ができる