

第3回:ラグランジュの運動方程式

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必要になる基礎知識

- テラー展開 $f(x+a) = \sum_{n=0}^{\infty} \frac{a^n}{n!} \frac{d^n f(x)}{dx^n} = f(x) + af'(x) + \frac{a^2}{2!} f''(x) + \frac{a^3}{3!} f'''(x) + \dots$
- テラー展開 $f(x+a, y+b) = f(x, y) + a \frac{\partial f(x, y)}{\partial x} + b \frac{\partial f(x, y)}{\partial y} + \frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial x \partial y}$ を用いる
- x_0 が $f(x)$ の極値を与えるとき、 $f(x)$ の満たす条件は？
- $\frac{d(AB)}{dt}$ の計算
- $\int_{t_i}^{t_f} \frac{d}{dt} f(t) dt$ の計算
- $\frac{d}{dt} \cos x(t)$ の計算

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ラグランジュの運動方程式の導出

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt$$

$x_1(t) = \text{本当とする}$ $x_2(t) = x_1(t) + \delta x(t)$

$$S(x_2, v_2) = \int_{t_i}^{t_f} \mathcal{L}(x_2(t), v_2(t)) dt = \int_{t_i}^{t_f} \mathcal{L}(x_1(t) + \delta x(t), v_1(t) + \delta v(t)) dt$$

最小作用の原理 $> S(x_1, v_1) = \int_{t_i}^{t_f} \mathcal{L}(x_1(t), v_1(t)) dt \Rightarrow$ 最小値を求める方法は?

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最小値を求める方法＝関数微分(復習)

関数: $f(t)$ の最小値 $\Rightarrow \frac{df(t)}{dt} = 0$

$f(t + \delta t) = f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2 + \frac{1}{3!} \frac{d^3 f(t)}{dt^3} (\delta t)^3 + (\dots)$

$$= \sum_{n=0}^{\infty} \frac{(\delta t)^n}{n!} \frac{d^n f(t)}{dt^n}$$
 (テラー展開)

$\delta t = \frac{1}{1,000} = 0.001$ (小数点以下表示桁数6桁) \Rightarrow **0と扱う**

$\Rightarrow (\delta t)^2 = \frac{1}{1,000,000} = 0.000001$ (小数点以下表示桁数6桁)

$(\delta t)^3 = \frac{1}{1,000,000,000} = 0.000000001$ (小数点以下表示桁数6桁)

$(\delta t)^4 = (\delta t)^5 = \dots = 0.000000000$ (小数点以下表示桁数6桁)

$$= f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2$$

$$f(t + \delta t) = f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2$$

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$$f(t + \delta t) = f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2$$

$\delta f(t) = f(t + \delta t) - f(t)$ とする...

$$\delta f(t) = \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2 \approx \frac{df(t)}{dt} \delta t = 0, \frac{d^2 f(t)}{dt^2} > 0$$
 してみる

$$= \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2 = \text{常に正} \Rightarrow \delta f(t) > 0 \Rightarrow f(t) \text{ は最小値}$$

$$\frac{df(t)}{dt} = 0, \frac{d^2 f(t)}{dt^2} > 0 \Rightarrow \frac{df(t)}{dt} = 0$$
 のみにして $\frac{d^2 f(t)}{dt^2} > 0$ は覚えておく

$$\frac{df(t)}{dt} = 0$$

$$\delta f(t) = f(t + \delta t) - f(t) = 0 \Leftrightarrow \frac{df(t)}{dt} = 0$$

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汎関数微分と最小値

最小作用の原理 $S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt$ の最小値 $= \mathcal{L}(x(t), v(t))$ の最小値 $\Rightarrow \delta \mathcal{L} = 0$

$\delta x(t) \approx 0, \delta v(t) \approx 0$

$$\delta \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) = \mathcal{L}(x(t), v(t)) + \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} \delta x(t) + \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \delta v(t)$$

$$+ \frac{1}{2} \left[\frac{\partial^2 \mathcal{L}(x(t), v(t))}{\partial x(t)^2} (\delta x(t))^2 + 2 \frac{\partial^2 \mathcal{L}(x(t), v(t))}{\partial x(t) \partial v(t)} \delta x(t) \delta v(t) + \frac{\partial^2 \mathcal{L}(x(t), v(t))}{\partial v(t)^2} (\delta v(t))^2 \right]$$

$$\delta \mathcal{L}(x(t), v(t)) = \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) - \mathcal{L}(x(t), v(t)) = \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \delta v(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} = 0$$

関数の形が変わる \Rightarrow 変分法

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最小値を求める

$$\delta \mathcal{L}(x(t), v(t)) = \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \delta v(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} = 0$$

$$x(t) = x_{\text{本}}(t) + \delta x(t) \Rightarrow v(t) = \frac{dx(t)}{dt} = \frac{d(x_{\text{本}}(t) + \delta x(t))}{dt} = \frac{dx_{\text{本}}(t)}{dt} + \frac{d\delta x(t)}{dt} \Rightarrow \delta v(t) = \frac{d\delta x(t)}{dt}$$

$$\begin{aligned} \delta \mathcal{L}(x(t), v(t)) &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \delta v(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \\ &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d\delta x(t)}{dt} \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \\ &\leq \frac{d}{dt}(A)B + A \frac{dB}{dt} - \frac{dA}{dt}B = \frac{d}{dt}(AB) - dB A \end{aligned}$$

$$\begin{aligned} \frac{d\delta x(t)}{dt} \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} &= \frac{d}{dt} \left(\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) - \delta x(t) \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \\ &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d}{dt} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] - \delta x(t) \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \end{aligned}$$

$$\begin{aligned} \delta \mathcal{L}(x(t), v(t)) &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d}{dt} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] - \delta x(t) \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \\ &\Rightarrow \delta S(x, v) = \int_{t_0}^{t_f} \delta \mathcal{L}(x(t), v(t)) dt = 0 \end{aligned}$$

$$\begin{aligned} 0 &= \delta S(x, v) = \int_{t_0}^{t_f} \delta \mathcal{L}(x(t), v(t)) dt \\ &= \int_{t_0}^{t_f} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d}{dt} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] - \delta x(t) \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt \\ &= \int_{t_0}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt + \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right]_{t_0}^{t_f} \\ &= \int_{t_0}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt + \delta x(t_f) \frac{\partial \mathcal{L}(x(t_f), v(t_f))}{\partial v(t_f)} - \delta x(t_0) \frac{\partial \mathcal{L}(x(t_0), v(t_0))}{\partial v(t_0)} \end{aligned}$$

ラグランジュの運動方程式

$$0 = \int_{t_0}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt + \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right]_{t_0}^{t_f}$$

$x_1(t_0) = x_1(t_0) \Rightarrow \frac{\partial x_1(t_0)}{\partial t} = 0$
 $x_2(t_f) = x_2(t_f) \Rightarrow \frac{\partial x_2(t_f)}{\partial t} = 0$

$\Rightarrow \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = 0$

ラグランジュの運動方程式

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)}$$

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ラグランジュの運動方程式のまとめ

$$S(x, v) = \int_{t_0}^{t_f} \mathcal{L}(x(t), v(t)) dt \text{ を最小の値にする時、} \begin{cases} x(t) = x_{\text{本}}(t) \\ v(t) = \frac{dx_{\text{本}}(t)}{dt} \end{cases}$$

$$S(x, v) = \int_{t_0}^{t_f} \mathcal{L}(x(t), v(t)) dt$$

$$S(x + \delta x, v + \delta v) = \int_{t_0}^{t_f} \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) dt$$

最小値を得るには... $\delta S(x, v) = 0$

$$\begin{aligned} \delta S(x, v) &= S(x + \delta x, v + \delta v) - S(x, v) \\ &= \int_{t_0}^{t_f} \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) dt - \int_{t_0}^{t_f} \mathcal{L}(x(t), v(t)) dt \\ &= \int_{t_0}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt = 0 \end{aligned}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} \Rightarrow \begin{cases} x(t) = x_{\text{本}}(t) \\ v(t) = \frac{dx_{\text{本}}(t)}{dt} \end{cases}$$

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一般座標の導入

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)}$$

$$\begin{cases} \text{直交座標: } q_1 = x, q_2 = y, q_3 = z \Rightarrow (x, y, z) \\ \text{円筒座標: } q_1 = r, q_2 = \theta, q_3 = z \Rightarrow (r, \theta, z) \\ \text{極座標: } q_1 = r, q_2 = \varphi, q_3 = \theta \Rightarrow (r, \varphi, \theta) \\ \vdots \end{cases} \Rightarrow \mathbf{q} = (q_1, q_2, q_3)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t))}{\partial \dot{\mathbf{q}}(t)} \right) = \frac{\partial \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t))}{\partial \mathbf{q}(t)} \leftarrow \begin{cases} \mathbf{q} = \text{一般座標} \\ \dot{\mathbf{q}} = \left(\frac{dq}{dt} \right) = \text{一般速度} \end{cases}$$

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3次元直交座標

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t))}{\partial \dot{\mathbf{q}}(t)} \right) = \frac{\partial \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t))}{\partial \mathbf{q}(t)} \quad \mathbf{q} = (x, y, z)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))}{\partial \dot{x}(t)} \right) = \frac{\partial \mathcal{L}(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))}{\partial x(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))}{\partial \dot{y}(t)} \right) = \frac{\partial \mathcal{L}(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))}{\partial y(t)}$$

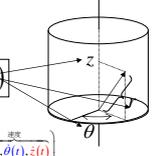
$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))}{\partial \dot{z}(t)} \right) = \frac{\partial \mathcal{L}(x(t), y(t), z(t), \dot{x}(t), \dot{y}(t), \dot{z}(t))}{\partial z(t)}$$

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3次元円筒座標

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial \dot{q}(t)} \right) = \frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)}$$

$$\mathbf{q} = (r, \theta, z)$$



$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(r(t), \theta(t), z(t), \dot{r}(t), \dot{\theta}(t), \dot{z}(t))}{\partial \dot{r}(t)} \right) = \frac{\partial \mathcal{L}(r(t), \theta(t), z(t), \dot{r}(t), \dot{\theta}(t), \dot{z}(t))}{\partial r(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(r(t), \theta(t), z(t), \dot{r}(t), \dot{\theta}(t), \dot{z}(t))}{\partial \dot{\theta}(t)} \right) = \frac{\partial \mathcal{L}(r(t), \theta(t), z(t), \dot{r}(t), \dot{\theta}(t), \dot{z}(t))}{\partial \theta(t)}$$

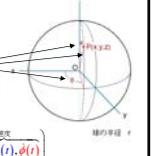
$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(r(t), \theta(t), z(t), \dot{r}(t), \dot{\theta}(t), \dot{z}(t))}{\partial \dot{z}(t)} \right) = \frac{\partial \mathcal{L}(r(t), \theta(t), z(t), \dot{r}(t), \dot{\theta}(t), \dot{z}(t))}{\partial z(t)}$$

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3次元極座標

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial \dot{q}(t)} \right) = \frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)}$$

$$\mathbf{q} = (r, \theta, \phi)$$



$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(r(t), \theta(t), \phi(t), \dot{r}(t), \dot{\theta}(t), \dot{\phi}(t))}{\partial \dot{r}(t)} \right) = \frac{\partial \mathcal{L}(r(t), \theta(t), \phi(t), \dot{r}(t), \dot{\theta}(t), \dot{\phi}(t))}{\partial r(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(r(t), \theta(t), \phi(t), \dot{r}(t), \dot{\theta}(t), \dot{\phi}(t))}{\partial \dot{\theta}(t)} \right) = \frac{\partial \mathcal{L}(r(t), \theta(t), \phi(t), \dot{r}(t), \dot{\theta}(t), \dot{\phi}(t))}{\partial \theta(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(r(t), \theta(t), \phi(t), \dot{r}(t), \dot{\theta}(t), \dot{\phi}(t))}{\partial \dot{\phi}(t)} \right) = \frac{\partial \mathcal{L}(r(t), \theta(t), \phi(t), \dot{r}(t), \dot{\theta}(t), \dot{\phi}(t))}{\partial \phi(t)}$$

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作用の見かけ上の違いと運動方程式

$$S(x, v) = \int_{t_1}^{t_2} \mathcal{L}(x(t), v(t)) dt \Rightarrow \delta S(x, v) = 0 \Rightarrow \text{運動方程式}$$

$$S(x, v) = \int_{t_1}^{t_2} \mathcal{L}(x(t), v(t)) dt, \quad S \left(\begin{matrix} x + \delta x \\ v + \delta v \end{matrix} \right) = \int_{t_1}^{t_2} \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) dt$$

$$\delta S(x, v) = S(x + \delta x, v + \delta v) - S(x, v) \quad \left[S \text{ と } S_{\text{new}} \text{ は見かけ上定数だけ違う} \right]$$

$$S_{\text{new}}(x, v) = S(x, v) + \text{定数}, \quad S_{\text{new}} \left(\begin{matrix} x + \delta x \\ v + \delta v \end{matrix} \right) = S(x + \delta x, v + \delta v) + \text{定数}$$

$$\delta S_{\text{new}}(x, v) = S_{\text{new}}(x + \delta x, v + \delta v) - S_{\text{new}}(x, v)$$

$$= S(x + \delta x, v + \delta v) + \text{定数} - [S(x, v) + \text{定数}] \Rightarrow \delta S_{\text{new}}(x, v) = \delta S(x, v)$$

$$\delta S(x, v) = 0 \text{ の解は自動的に } \delta S_{\text{new}}(x, v) = 0 \text{ の解になる}$$

$$S(x, v) \text{ と } S_{\text{new}}(x, v) = S(x, v) + \text{定数} \text{ は同じ運動方程式を与える}$$

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ラグランジュ関数の見かけ上の違いと運動方程式

$$S(x, v) = \int_{t_1}^{t_2} \mathcal{L}(x(t), v(t)) dt, \quad S_{\text{new}}(x, v) = \int_{t_1}^{t_2} \mathcal{L}_{\text{new}}(x(t), v(t)) dt = S(x, v) + \text{定数}$$

$$S_{\text{new}}(x, v) = \int_{t_1}^{t_2} \mathcal{L}_{\text{new}}(x(t), v(t)) dt = \int_{t_1}^{t_2} \mathcal{L}(x(t), v(t)) dt + \int_{t_1}^{t_2} f(x(t), v(t)) dt$$

【公式】 $\int_{t_1}^{t_2} \frac{dA(x(t), v(t))}{dt} dt = [A(x(t), v(t))]_{t_1}^{t_2} = A(x(t_2), v(t_2)) - A(x(t_1), v(t_1)) = \text{定数}$

$$f(x(t), v(t)) = \frac{dA(x(t), v(t))}{dt} \text{ なら自動的に } \int_{t_1}^{t_2} f(x(t), v(t)) dt \text{ は定数になる}$$

$$= \int_{t_1}^{t_2} \mathcal{L}(x(t), v(t)) dt + \int_{t_1}^{t_2} \frac{dA(x(t), v(t))}{dt} dt$$

$$\mathcal{L}_{\text{new}}(x(t), v(t)) = \mathcal{L}(x(t), v(t)) + \frac{dA(x(t), v(t))}{dt}$$

$$\mathcal{L}(x(t), v(t)) \text{ と } \mathcal{L}_{\text{new}}(x(t), v(t)) \text{ は同じ運動方程式を与える}$$

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ラグランジュ関数の見かけ上の違いと新力学???

新たなニュートン力学: 安江力学発見か?

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \frac{1}{2} m [v(t)]^2 - U(x(t)) + \dot{Y} v(t) \cos x(t)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial x(t)}$$

$$\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial v(t)} = \frac{\partial}{\partial v(t)} \left(\frac{1}{2} m [v(t)]^2 - U(x(t)) + Y v(t) \cos x(t) \right) = mv(t) + Y \cos x(t)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial v(t)} \right) = m \frac{dv(t)}{dt} + Y \frac{d \cos x(t)}{dt} = ma(t) - Y v(t) \sin x(t)$$

【公式】 $\frac{d \cos x(t)}{dt} = \frac{dx(t)}{dt} \frac{d \cos x(t)}{dx(t)} = v(t) (-\sin x(t)) = -v(t) \sin x(t)$

$$\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial x(t)} = \frac{\partial}{\partial x(t)} \left(\frac{1}{2} m [v(t)]^2 - U(x(t)) + Y v(t) \cos x(t) \right) = -\frac{\partial U(x(t))}{\partial x(t)} + Y v(t) (-\sin x(t))$$

$$ma(t) - Y v(t) \sin x(t) = -\frac{\partial U(x(t))}{\partial x(t)} - Y v(t) \sin x(t)$$

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ラグランジュ関数の見かけ上の違い(続き)

$$ma(t) - Y v(t) \sin x(t) = -\frac{\partial U(x(t))}{\partial x(t)} - Y v(t) \sin x(t)$$

$$\Rightarrow ma(t) = -\frac{\partial U(x(t))}{\partial x(t)} \Rightarrow \text{ニュートンの運動方程式}$$

ニュートン力学じゃん!!! 何で???

同じ力学になる場合

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \mathcal{L}_{\text{ニュートン}}(x(t), v(t)) + \frac{dA(x(t), v(t))}{dt}$$

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \mathcal{L}_{\text{ニュートン}}(x(t), v(t)) + \dot{Y} v(t) \cos x(t)$$

$$\llcorner v(t) \cos x(t) \llcorner = \frac{d}{dt} \left(\int \cos x(t) dx(t) \right) = \frac{d}{dt} \sin x(t)$$

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \mathcal{L}_{\text{ニュートン}}(x(t), v(t)) + \frac{d}{dt} [Y \sin x(t)]$$

$$A(x(t), v(t)) = Y \sin x(t) \Rightarrow \text{新しい力学は出ない!!!}$$

N粒子のラグランジュ関数

$$S(x_1, x_2, \dots, x_N, v_1, v_2, \dots, v_N) = \int_{t_i}^{t_f} \mathcal{L} \left(\overbrace{x_1(t), x_2(t), \dots, x_N(t)}^{N \text{ 粒子の位置}}, \overbrace{v_1(t), v_2(t), \dots, v_N(t)}^{N \text{ 粒子の速度}} \right) dt$$

簡潔に表記するとき
=ベクトル表記

$$S(\mathbf{x}, \mathbf{v}) = \int_{t_i}^{t_f} \mathcal{L} \left(\overbrace{\mathbf{x}(t)}^{N \text{ 粒子の位置}}, \overbrace{\mathbf{v}(t)}^{N \text{ 粒子の速度}} \right) dt \quad \left(\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt} \right)$$

N粒子の運動方程式

$$\left. \begin{aligned} \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x_1(t), v_1(t))}{\partial v_1(t)} \right) &= \frac{\partial \mathcal{L}(x_1(t), v_1(t))}{\partial x_1(t)} \\ \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x_2(t), v_2(t))}{\partial v_2(t)} \right) &= \frac{\partial \mathcal{L}(x_2(t), v_2(t))}{\partial x_2(t)} \\ &\vdots \\ \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x_N(t), v_N(t))}{\partial v_N(t)} \right) &= \frac{\partial \mathcal{L}(x_N(t), v_N(t))}{\partial x_N(t)} \end{aligned} \right\} \frac{d}{dt} \left(\frac{\partial \mathcal{L}(\mathbf{x}(t), \mathbf{v}(t))}{\partial \mathbf{v}(t)} \right) = \frac{\partial \mathcal{L}(\mathbf{x}(t), \mathbf{v}(t))}{\partial \mathbf{x}(t)}$$

計算するとき

簡潔に表記するとき