

第3回：ラグランジュの運動方程式

必要になる基礎知識

○テーラー展開 $f(x+a) = \sum_{n=0}^{\infty} \frac{a^n}{n!} \frac{d^n f(x)}{dx^n} = f(x) + af'(x) + \frac{a^2}{2!} f''(x) + \frac{a^3}{3!} f'''(x) + \dots$

○テーラー展開 $f(x+a, y+b) = f(x, y) + a \frac{\partial f(x, y)}{\partial x} + b \frac{\partial f(x, y)}{\partial y} + \boxed{\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial x \partial y}}$ を用いる

○ x_0 が $f(x)$ の極値を与えるとき、 $f(x)$ の満たす条件は？

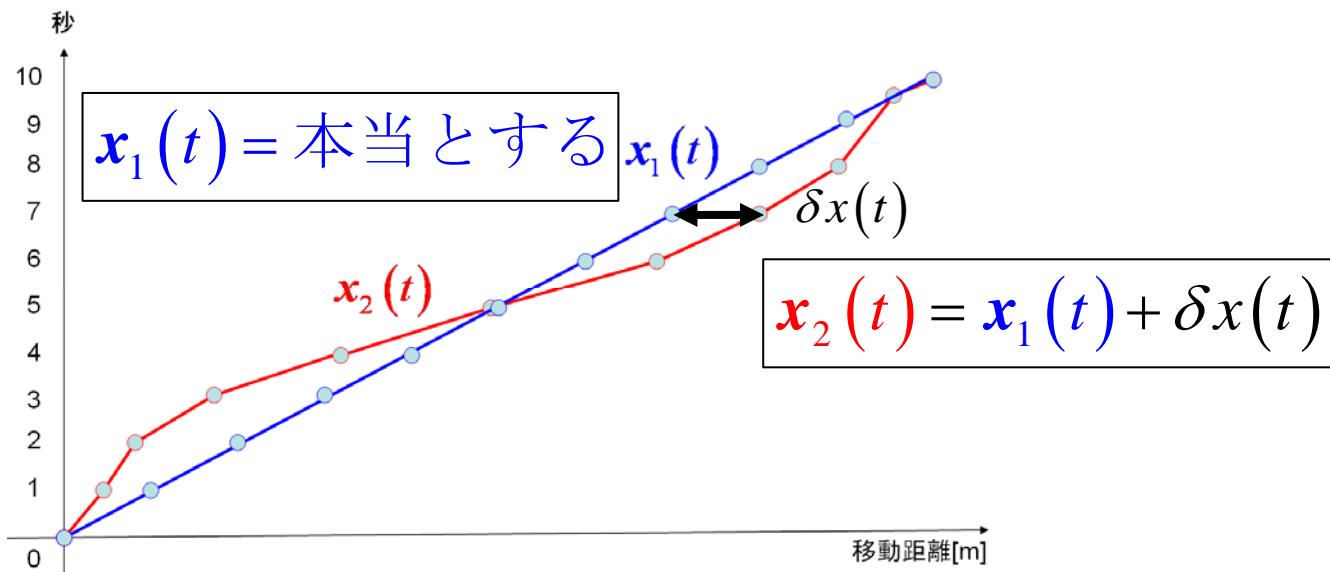
○ $\frac{d(AB)}{dt}$ の計算

○ $\int_{t_i}^{t_f} \frac{d}{dt} f(t) dt$ の計算

○ $\frac{d}{dt} \cos x(t)$ の計算

ラグランジュの運動方程式の導出

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt$$



$$S(x_2, v_2) = \int_{t_i}^{t_f} \mathcal{L}(x_2(t), v_2(t)) dt = \int_{t_i}^{t_f} \mathcal{L}(x_1(t) + \delta x(t), v_1(t) + \delta v(t)) dt$$

最小作用の原理

$$> S(x_1, v_1) = \int_{t_i}^{t_f} \mathcal{L}(x_1(t), v_1(t)) dt \Rightarrow \boxed{\text{最小値を求める方法は?}}$$

最小値を求める方法=関数微分(復習)

$$\boxed{\text{関数} : f(t) \text{の最小値} \Rightarrow \frac{df(t)}{dt} = 0}$$

$$f(t + \delta t) = f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2 + \frac{1}{3!} \cancel{\frac{d^3 f(t)}{dt^3}} (\delta t)^3 + \cdots + (\infty \text{まで} \cancel{\text{続く}})$$

$$= \sum_{n=0}^{\infty} \frac{(\delta t)^n}{n!} \frac{d^n f(t)}{dt^n} \quad (\text{テーラー展開})$$

$$\delta t = \frac{1}{1,000} = 0.001 \begin{pmatrix} \text{小数点以下} \\ \text{表示桁数6桁} \end{pmatrix} \quad \boxed{\delta t \neq 0} \text{ と表す}$$

$$\Rightarrow (\delta t)^2 = \frac{1}{1,000,000} = 0.000001 \begin{pmatrix} \text{小数点以下} \\ \text{表示桁数6桁} \end{pmatrix}$$

$$(\delta t)^3 = \frac{1}{1,000,000,000} = 0.000000 \begin{pmatrix} \text{小数点以下} \\ \text{表示桁数6桁} \end{pmatrix}$$

$$(\delta t)^4 = (\delta t)^5 = \cdots = 0.000000 \begin{pmatrix} \text{小数点以下} \\ \text{表示桁数6桁} \end{pmatrix}$$

$$= f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2$$

$$\boxed{f(t + \delta t) = f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2}$$

$$f(t + \delta t) = f(t) + \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2$$

$\delta f(t) = f(t + \delta t) - f(t)$ とする・・・

$$\delta f(t) = \overbrace{f(t + \delta t)}^{\text{最小値よりいつも大きい}} - \overbrace{f(t)}^{\text{最小値}} > 0 \quad (\text{最小値の条件})$$

$$\delta f(t) = \frac{df(t)}{dt} \delta t + \frac{1}{2!} \frac{d^2 f(t)}{dt^2} (\delta t)^2 \leftarrow \frac{df(t)}{dt} = 0, \frac{d^2 f(t)}{dt^2} > 0 \text{にしてみる}$$

$$= \frac{1}{2!} \overbrace{\frac{d^2 f(t)}{dt^2}}^{\text{正にする}} (\delta t)^2 = \text{常に正} \Rightarrow \delta f(t) > 0 \Rightarrow f(t) \text{は最小値}$$

$$\boxed{\frac{df(t)}{dt} = 0, \frac{d^2 f(t)}{dt^2} > 0} \Rightarrow \boxed{\frac{df(t)}{dt} = 0} \text{のみにして } \frac{d^2 f(t)}{dt^2} > 0 \text{は覚えておく}$$

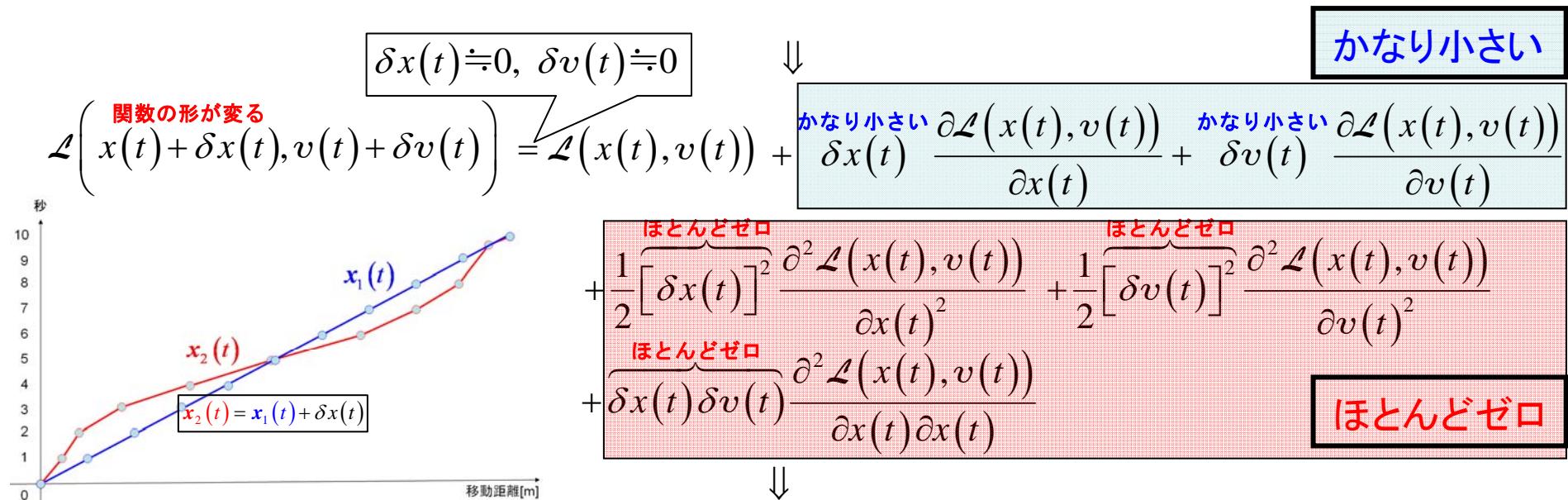
$$\boxed{\frac{df(t)}{dt} = 0}$$



$$\delta f(t) = f(t + \delta t) - f(t) = 0 \Leftrightarrow \frac{df(t)}{dt} = 0$$

汎関数微分と最小値

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt \quad \overbrace{\text{最小値}}^{\text{最小作用の原理}} = \mathcal{L}\left(\overbrace{x(t)}^{\text{関数}}, \overbrace{v(t)}^{\text{関数}}\right) \text{の最小値} \Rightarrow \delta \mathcal{L} = 0$$



$$\begin{aligned} \delta \mathcal{L}(x(t), v(t)) &= \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) - \mathcal{L}(x(t), v(t)) \\ &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \delta v(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \underset{\text{最小値}}{=} 0 \end{aligned}$$

関数の形が変わる \Rightarrow 变分法

最小値を求める

$$\delta \mathcal{L}(x(t), v(t)) = \delta x(t) \overbrace{\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)}}^{\text{汎関数微分}} + \delta v(t) \overbrace{\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)}}^{\text{汎関数微分}} = 0$$

$$x(t) = x_{\text{本当}}(t) + \delta x(t) \Rightarrow v(t) = \frac{dx(t)}{dt} = \frac{d(x_{\text{本当}}(t) + \delta x(t))}{dt} = \frac{dx_{\text{本当}}(t)}{dt} + \frac{d\delta x(t)}{dt}$$

$$v(t) = v_{\text{本当}}(t) + \delta v(t)$$

$$\delta v(t) = \frac{d\delta x(t)}{dt}$$

$$\begin{aligned} \delta \mathcal{L}(x(t), v(t)) &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \delta v(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \\ &= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \boxed{\frac{d\delta x(t)}{dt} \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)}} \\ &\Leftarrow \frac{d}{dt}(AB) = \frac{dA}{dt}B + A\frac{dB}{dt} \Rightarrow \boxed{\frac{dA}{dt}B} = \frac{d}{dt}(AB) - \frac{dB}{dt}A \end{aligned}$$

$A = \delta x(t)$
 $B = \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)}$

$$\boxed{\frac{d\delta x(t)}{dt} \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)}} = \frac{d}{dt} \left(\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) - \delta x(t) \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right)$$

$$= \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d}{dt} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] - \delta x(t) \frac{d}{dt} \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right]$$

$$\delta \mathcal{L}(x(t), v(t)) = \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d}{dt} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] - \delta x(t) \frac{d}{dt} \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right]$$

$$\Rightarrow \delta S(x, v) = \int_{t_i}^{t_f} \delta \mathcal{L}(x(t), v(t)) dt = 0$$

$$0 = \delta S(x, v) = \int_{t_i}^{t_f} \delta \mathcal{L}(x(t), v(t)) dt$$

$$= \int_{t_i}^{t_f} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} + \frac{d}{dt} \left[\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] - \delta x(t) \frac{d}{dt} \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right] \right] dt$$

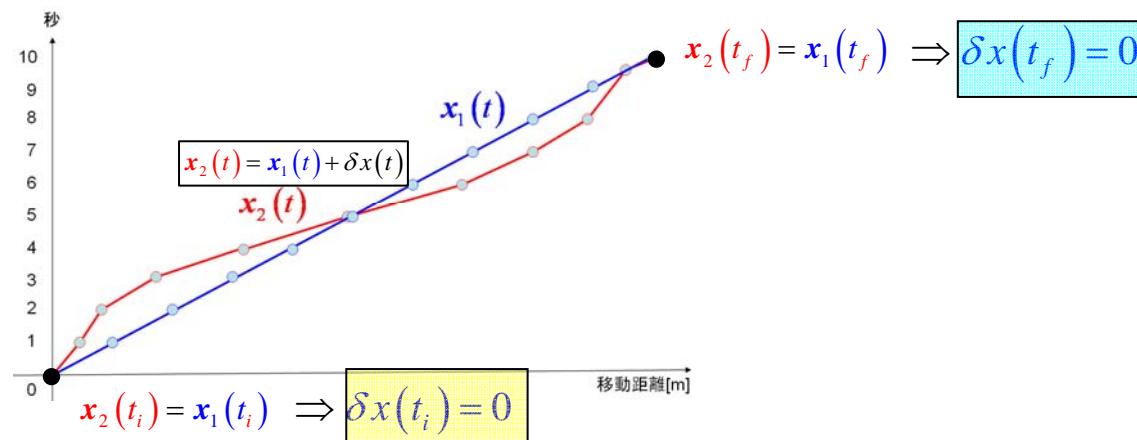
$$= \int_{t_i}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt + \boxed{\int_{t_i}^{t_f} \frac{d}{dt} \left(\delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) dt}$$


$$\begin{aligned} & \Leftarrow \boxed{\int_{t_i}^{t_f} \frac{d}{dt} f(t) dt} = f(t_f) - f(t_i) \quad f(t) = \delta x(t) \frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \\ & = \delta x(t_f) \frac{\partial \mathcal{L}(x(t_f), v(t_f))}{\partial v(t_f)} - \delta x(t_i) \frac{\partial \mathcal{L}(x(t_i), v(t_i))}{\partial v(t_i)} \end{aligned}$$

$$= \int_{t_i}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt + \delta x(t_f) \frac{\partial \mathcal{L}(x(t_f), v(t_f))}{\partial v(t_f)} - \delta x(t_i) \frac{\partial \mathcal{L}(x(t_i), v(t_i))}{\partial v(t_i)}$$

ラグランジュの運動方程式

$$0 = \int_{t_i}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt + \boxed{\delta x(t_f) \frac{\partial \mathcal{L}(x(t_f), v(t_f))}{\partial v(t_f)} = 0} - \boxed{\delta x(t_i) \frac{\partial \mathcal{L}(x(t_i), v(t_i))}{\partial v(t_i)} = 0}$$



$$0 = \int_{t_i}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt$$

⇐ どんな $\delta x(t)$ でもゼロになる

$$\Rightarrow \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = 0$$

ラグランジュの運動方程式

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)}$$

ラグランジュの運動方程式のまとめ

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt$$

を最小の値にする時、

$$\begin{cases} x(t) = x_{\text{本当}}(t) \\ v(t) = \frac{dx_{\text{本当}}(t)}{dt} \end{cases}$$

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt$$

$$S(x + \delta x, v + \delta v) = \int_{t_i}^{t_f} \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) dt$$

最小値を得るには・・・

$$\boxed{\delta S(x, v) = 0}$$

$$\begin{aligned} \delta S(x, v) &= S(x + \delta x, v + \delta v) - S(x, v) \\ &= \int_{t_i}^{t_f} \mathcal{L}(x(t) + \delta x(t), v(t) + \delta v(t)) dt - \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt \\ &= \int_{t_i}^{t_f} \delta x(t) \left[\frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) \right] dt = 0 \end{aligned}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}(x(t), v(t))}{\partial x(t)} \Rightarrow \begin{cases} x(t) = x_{\text{本当}}(t) \\ v(t) = \frac{dx_{\text{本当}}(t)}{dt} \end{cases}$$

一般座標の導入

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L} \left(\overbrace{x(t)}^{\text{直交座標系}}, v(t) \right)}{\partial x(t)}$$

$$\left. \begin{array}{l} \text{直交座標 : } q_1 = x, q_2 = y, q_3 = z \Rightarrow (x, y, z) \\ \text{円筒座標 : } q_1 = r, q_2 = \theta, q_3 = z \Rightarrow (r, \theta, z) \\ \text{極座標 : } q_1 = r, q_2 = \varphi, q_3 = \theta \Rightarrow (r, \varphi, \theta) \\ \vdots \end{array} \right\} \Rightarrow \mathbf{q} = (q_1, q_2, q_3)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t))}{\partial \mathbf{q}(t)} \right) = \frac{\partial \mathcal{L}(\mathbf{q}(t), \dot{\mathbf{q}}(t))}{\partial \dot{\mathbf{q}}(t)} \leftarrow \begin{cases} \mathbf{q} = \text{一般座標} \\ \dot{\mathbf{q}} \left(= \frac{d\mathbf{q}}{dt} \right) = \text{一般速度} \end{cases}$$

3次元直交座標

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)} \right) = \frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)} \quad q = (x, y, z)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}\left(x(t), \underbrace{y(t), z(t)}_{\text{速度}}, \overbrace{\dot{x}(t), \dot{y}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \dot{x}(t)} \right) = \frac{\partial \mathcal{L}\left(x(t), y(t), z(t), \overbrace{\dot{x}(t), \dot{y}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial x(t)}$$

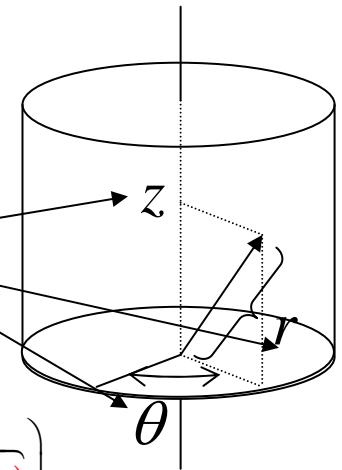
$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}\left(x(t), \underbrace{y(t), z(t)}_{\text{速度}}, \overbrace{\dot{x}(t), \dot{y}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \dot{y}(t)} \right) = \frac{\partial \mathcal{L}\left(x(t), y(t), z(t), \overbrace{\dot{x}(t), \dot{y}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial y(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}\left(x(t), \underbrace{y(t), z(t)}_{\text{速度}}, \overbrace{\dot{x}(t), \dot{y}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \dot{z}(t)} \right) = \frac{\partial \mathcal{L}\left(x(t), y(t), z(t), \overbrace{\dot{x}(t), \dot{y}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial z(t)}$$

3次元円筒座標

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)} \right) = \frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)}$$

$$q = (r, \theta, z)$$



$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}\left(r(t), \theta(t), z(t), \overbrace{\dot{r}(t), \dot{\theta}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \dot{r}(t)} \right) = \frac{\partial \mathcal{L}\left(r(t), \theta(t), z(t), \overbrace{\dot{r}(t), \dot{\theta}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial r(t)}$$

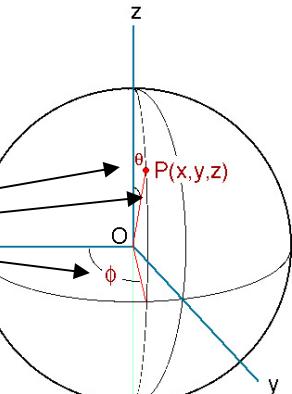
$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}\left(r(t), \theta(t), z(t), \overbrace{\dot{r}(t), \dot{\theta}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \dot{\theta}(t)} \right) = \frac{\partial \mathcal{L}\left(r(t), \theta(t), z(t), \overbrace{\dot{r}(t), \dot{\theta}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \theta(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}\left(r(t), \theta(t), z(t), \overbrace{\dot{r}(t), \dot{\theta}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial \dot{z}(t)} \right) = \frac{\partial \mathcal{L}\left(r(t), \theta(t), z(t), \overbrace{\dot{r}(t), \dot{\theta}(t), \dot{z}(t)}^{\text{速度}}\right)}{\partial z(t)}$$

3次元極座標

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)} \right) = \frac{\partial \mathcal{L}(q(t), \dot{q}(t))}{\partial q(t)}$$

$$q = (r, \theta, \phi)$$



球の半径 r

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L} \left(r(t), \theta(t), \phi(t), \overset{\text{速度}}{\dot{r}(t)}, \overset{\text{速度}}{\dot{\theta}(t)}, \overset{\text{速度}}{\dot{\phi}(t)} \right)}{\partial \overset{\text{速度}}{\dot{r}(t)}} \right) = \frac{\partial \mathcal{L} \left(r(t), \theta(t), \phi(t), \overset{\text{速度}}{\dot{r}(t)}, \overset{\text{速度}}{\dot{\theta}(t)}, \overset{\text{速度}}{\dot{\phi}(t)} \right)}{\partial r(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L} \left(r(t), \theta(t), \phi(t), \overset{\text{速度}}{\dot{r}(t)}, \overset{\text{速度}}{\dot{\theta}(t)}, \overset{\text{速度}}{\dot{\phi}(t)} \right)}{\partial \overset{\text{速度}}{\dot{\theta}(t)}} \right) = \frac{\partial \mathcal{L} \left(r(t), \theta(t), \phi(t), \overset{\text{速度}}{\dot{r}(t)}, \overset{\text{速度}}{\dot{\theta}(t)}, \overset{\text{速度}}{\dot{\phi}(t)} \right)}{\partial \theta(t)}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L} \left(r(t), \theta(t), \phi(t), \overset{\text{速度}}{\dot{r}(t)}, \overset{\text{速度}}{\dot{\theta}(t)}, \overset{\text{速度}}{\dot{\phi}(t)} \right)}{\partial \overset{\text{速度}}{\dot{\phi}(t)}} \right) = \frac{\partial \mathcal{L} \left(r(t), \theta(t), \phi(t), \overset{\text{速度}}{\dot{r}(t)}, \overset{\text{速度}}{\dot{\theta}(t)}, \overset{\text{速度}}{\dot{\phi}(t)} \right)}{\partial \phi(t)}$$

作用の見かけ上の違いと運動方程式

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt \Rightarrow \delta S(x, v) = 0 \Rightarrow \text{運動方程式}$$

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt, \quad S\left(\begin{array}{c} \text{関数の形が変る} \\ x + \delta x, \quad v + \delta v \end{array}\right) = \int_{t_i}^{t_f} \mathcal{L}\left(\begin{array}{c} \text{関数の形が変る} \\ x(t) + \delta x(t), v(t) + \delta v(t) \end{array}\right) dt$$

$$\delta S(x, v) = S(x + \delta x, v + \delta v) - S(x, v)$$

S と S_{new} は見かけ上定数だけ違う

$$S_{\text{NEW}}(x, v) = S(x, v) + \text{定数}, \quad S_{\text{NEW}}\left(\begin{array}{c} \text{関数の形が変る} \\ x + \delta x, \quad v + \delta v \end{array}\right) = S(x + \delta x, v + \delta v) + \text{定数}$$

$$\delta S_{\text{new}}(x, v) = S_{\text{new}}(x + \delta x, v + \delta v) - S_{\text{new}}(x, v)$$

$$= S(x + \delta x, v + \delta v) + \text{定数} - [S(x, v) + \text{定数}]$$

$$= S(x + \delta x, v + \delta v) - S(x, v)$$

$$= \delta S(x, v)$$



$$\delta S_{\text{new}}(x, v) = \delta S(x, v)$$

$\delta S(x, v) = 0$ の解は自動的に $\delta S_{\text{new}}(x, v) = 0$ の解になる

$S(x, v)$ と $S_{\text{new}}(x, v) = S(x, v) + \text{定数}$ は同じ運動方程式を与える

どちらも $x(t) = x_{\text{本当}}(t), v(t) = \frac{dx_{\text{本当}}(t)}{dt}$ になる

ラグランジュ関数の見かけ上の違いと運動方程式

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt \quad S_{\text{new}}(x, v) = \int_{t_i}^{t_f} \mathcal{L}_{\text{new}}(x(t), v(t)) dt = S(x, v) + \text{定数}$$

$$\mathcal{L}_{\text{new}}(x(t), v(t)) = \mathcal{L}(x(t), v(t)) + f(x(t), v(t))$$

$$S_{\text{new}}(x, v) = \int_{t_i}^{t_f} \mathcal{L}_{\text{new}}(x(t), v(t)) dt = \overbrace{\int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt}^{S(x, v)} + \overbrace{\int_{t_i}^{t_f} f(x(t), v(t)) dt}^{\text{定数になるべし！！！}}$$

【公式】 $\int_{t_i}^{t_f} \frac{dA(x(t), v(t))}{dt} dt = [A(x(t), v(t))]_{t_i}^{t_f} = \overbrace{A(x(t_f), v(t_f))}^{t_f (=10\text{秒目})=\text{定数}} - \overbrace{A(x(t_i), v(t_i))}^{t_i (=0\text{秒目})=\text{定数}} = \text{定数}$

$f(x(t), v(t)) = \frac{dA(x(t), v(t))}{dt}$ なら **自動的に** $\int_{t_i}^{t_f} f(x(t), v(t)) dt$ は **定数**になる

$$= \overbrace{\int_{t_i}^{t_f} \mathcal{L}(x(t), v(t)) dt}^{S(x, v)} + \overbrace{\int_{t_i}^{t_f} \frac{dA(x(t), v(t))}{dt} dt}^{\text{自動的に定数になる！！！}}$$

$$\mathcal{L}_{\text{new}}(x(t), v(t)) = \mathcal{L}(x(t), v(t)) + \frac{dA(x(t), v(t))}{dt}$$

$\mathcal{L}(x(t), v(t))$ と $\mathcal{L}_{\text{new}}(x(t), v(t))$ は **同じ運動方程式**を与える ¹⁶

ラグランジュ関数の見かけ上の違いと 新力学？？？

新たなニュートン力学：安江力学発見か？

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \overbrace{\frac{1}{2}m[v(t)]^2 - U(x(t))}^{\substack{\text{ニュートン力学} \\ \text{安江定数}}} + \boxed{\begin{array}{ll} Y & \text{安江効果} \\ v(t)\cos x(t) & \end{array}}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial v(t)} \right) = \frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial x(t)}$$

$$\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial v(t)} = \frac{\partial}{\partial v(t)} \left(\frac{1}{2}m[v(t)]^2 - U(x(t)) + Yv(t)\cos x(t) \right) = m\color{blue}{v(t)} + Y\cos x(t)$$

$$\Rightarrow \frac{d}{dt} \left(\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial x(t)} \right) = m \frac{dv(t)}{dt} - Y \boxed{\frac{d \cos x(t)}{dt}} = ma(t) - Yv(t)\sin x(t)$$

【公式】 $\boxed{\frac{d \cos x(t)}{dt}} = \frac{dx(t)}{dt} \frac{d \cos x(t)}{dx(t)} = \color{red}{v(t)} (\color{blue}{-\sin x(t)}) = -v(t)\sin x(t)$

$$\frac{\partial \mathcal{L}_{\text{安江}}(x(t), v(t))}{\partial x(t)} = \frac{\partial}{\partial x(t)} \left(\frac{1}{2}m[v(t)]^2 - U(\color{red}{x(t)}) + Yv(t)\color{blue}{\cos x(t)} \right) = -\frac{\partial U(\color{red}{x(t)})}{\partial x(t)} + Yv(t)(-\sin x(t))$$

$$\boxed{ma(t) - Yv(t)\sin x(t) = -\frac{\partial U(x(t))}{\partial x(t)} - Yv(t)\sin x(t)}$$

ラグランジュ関数の見かけ上の違い(続き)

$$ma(t) - [Yv(t)\sin x(t)] = -\frac{\partial U(x(t))}{\partial x(t)} - [Yv(t)\sin x(t)]$$

$$\Rightarrow ma(t) = -\frac{\partial U(x(t))}{\partial x(t)} \Rightarrow \text{ニュートンの運動方程式}$$

ニュートン力学じゃん！！！ 何で？？？

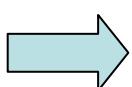
同じ力学になる場合

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \mathcal{L}_{\text{ニュートン}}(x(t), v(t)) + \frac{dA(x(t), v(t))}{dt}$$

$$\begin{aligned} \mathcal{L}_{\text{安江}}(x(t), v(t)) &= \mathcal{L}_{\text{ニュートン}}(x(t), v(t)) + \overset{\text{安江定数}}{\tilde{Y}} v(t) \cos x(t) \\ &\Leftarrow v(t) \cos x(t) = \frac{dx(t)}{dt} \cos x(t) = \frac{d}{dt} \sin x(t) \end{aligned}$$

$$\mathcal{L}_{\text{安江}}(x(t), v(t)) = \mathcal{L}_{\text{ニュートン}}(x(t), v(t)) + \frac{d[Y \sin x(t)]}{dt}$$

$$A(x(t), v(t)) = Y \sin x(t)$$



新しい力学は出ない！！！

N粒子のラグランジュ関数

$$S(x_1, x_2, \dots, x_N, v_1, v_2, \dots, v_N) = \int_{t_i}^{t_f} \mathcal{L} \left(\overbrace{x_1(t), x_2(t), \dots, x_N(t)}^{\text{N粒子の位置}}, \overbrace{v_1(t), v_2(t), \dots, v_N(t)}^{\text{N粒子の速度}} \right) dt$$

簡潔に表記するとき
=ベクトル表記

$$S(x, v) = \int_{t_i}^{t_f} \mathcal{L} \left(\overbrace{\mathbf{x}(t)}^{\text{N粒子の位置}}, \overbrace{\mathbf{v}(t)}^{\text{N粒子の速度}} \right) dt \quad \left(\mathbf{v}(t) = \frac{d\mathbf{x}(t)}{dt} \right)$$

N粒子の運動方程式

$$\left. \begin{aligned} \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x_1(t), v_1(t))}{\partial v_1(t)} \right) &= \frac{\partial \mathcal{L}(x_1(t), v_1(t))}{\partial x_1(t)} \\ \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x_2(t), v_2(t))}{\partial v_2(t)} \right) &= \frac{\partial \mathcal{L}(x_2(t), v_2(t))}{\partial x_2(t)} \\ &\vdots \\ \frac{d}{dt} \left(\frac{\partial \mathcal{L}(x_N(t), v_N(t))}{\partial v_N(t)} \right) &= \frac{\partial \mathcal{L}(x_N(t), v_N(t))}{\partial x_N(t)} \end{aligned} \right\} \quad \begin{aligned} \frac{d}{dt} \left(\frac{\partial \mathcal{L}(\mathbf{x}(t), \mathbf{v}(t))}{\partial \mathbf{v}(t)} \right) &= \frac{\partial \mathcal{L}(\mathbf{x}(t), \mathbf{v}(t))}{\partial \mathbf{x}(t)} \end{aligned}$$

計算するとき

簡潔に表記するとき